Max-Min Fairness Linear Transceiver Design for a Multi-User MIMO Interference Channel

Ya-Feng Liu, Yu-Hong Dai, and Zhi-Quan Luo

Abstract

Consider the max-min fairness linear transceiver design problem for a multi-user multi-input multi-output (MIMO) interference channel. When the channel knowledge is perfectly known, this problem can be formulated as the maximization of the minimum signal to interference plus noise ratio (SINR) utility, subject to individual power constraints at each transmitter. We prove in this paper that, if the number of antennas is at least two at each transmitter (receiver) and is at least three at each receiver (transmitter), the max-min fairness linear transceiver design problem is computationally intractable as the number of users becomes large. In fact, even the problem of checking the feasibility of a given set of target SINR levels is strongly NP-hard. We then propose two iterative algorithms to solve the max-min fairness linear transceiver design problem. The transceivers generated by these algorithms monotonically improve the min-rate utility and are guaranteed to converge to a stationary solution. The efficiency and performance of the proposed algorithms compare favorably with solutions obtained from the channel matched beamforming or the leakage interference minimization.

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Index Terms

Beamforming, complexity, max-min fairness, MIMO interference channel, transceiver design.

I. Introduction

With the rapid growth of wireless data traffic, multi-user interference has become a major bottleneck that limits the performance of the existing wireless communication services. Mathematically, we can model such an interference-limited communication system as a multi-user interference channel in which a number of linearly interfering transmitters simultaneously send private data to their respective receivers. When the transmitters and receivers are equipped with multiple antennas, one effective approach to mitigate multi-user interference is to jointly optimize the physical layer transmit-receive beamforming strategies for all users, subject to individual transmit power budget constraints. In general, the objective of joint transceiver optimization is to maximize a certain system utility [1]–[12] such as the sum-rate utility, the proportional fairness utility, among others.

The problem of system utility maximization for a MIMO interference channel has been a subject of intensive research in recent years. For instance, the sum-rate utility maximization problem has been shown to be NP-hard [9] and various algorithms have been proposed to find a local optimal solution [8], [10], [11]. In particular, an iteratively weighted minimum mean square error (WMMSE) approach was proposed in [11] to maximize the sum-rate utility for the MIMO interfering broadcast channel. This algorithm also works for the harmonic mean utility and the proportional fairness utility. For the latter two utilities, the corresponding problem has been shown to be NP-hard even for cases involving three or more tones or for a multi-input single-output (MISO) interference channel [9], [10].

In this paper, we design beamforming strategies by maximizing the min-rate utility (or equivalently the minimum SINR utility). This approach places the highest emphasis on the user fairness. This max-min fairness linear transceiver design has been studied in [1], [2] where the authors proposed to approximate the optimum by minimizing the sum of equally weighted inverse signal to interference ratios (SIR). For the single receive antenna case, the authors of [3] further extended this approach by choosing suitable weight factors with which the weighted sum of inverse SIR maximization can achieve optimal max-min fairness. Also, polynomial time algorithms capable of achieving global optimality have been proposed in [4] (also see [9], [10]) for the power control and/or transmit beamforming design problems, again for the single receive antenna case. These results imply that the max-min SINR precoder design problem with *fixed* receive beamformers can be solved in polynomial time [4]. Recently, it was shown in [13] that the max-min beamforming design problem for the single-input multi-output (SIMO) interference channel, where each transmitter is equipped with a single antenna, is polynomial time solvable. When there are more than one receive antenna and one transmit antenna per user, the max-min fairness linear transceiver design problem is not known to be solvable in polynomial time nor is it known to be NP-hard.

The contributions of this paper are twofold. First, we characterize the computational complexity of the max-min fairness linear transceiver design problem for a multi-user MIMO interference channel. We show that this problem is strongly NP-hard when the number of antennas is at least two at each transmitter (receiver) and is at least three at each receiver (transmitter). In fact, even the problem of checking the feasibility of a given set of target SINR levels is strongly NP-hard. This intractability result is in contrast to the polynomial time solvability of the max-min SINR beamforming problem in the MISO (or SIMO) case (i.e., the single receive or transmit antenna case); see [4], [9], [10], [13], [14]. Second, we propose two efficient algorithms to design linear transceivers according to the max-min fairness criterion. One algorithm, called exact cyclic coordinate ascent algorithm (ECCAA), is based on the exact cyclic coordinate ascent strategy, while the other algorithm, called inexact cyclic coordinate ascent algorithm (ICCAA), uses an *inexact* cyclic coordinate ascent strategy. These two cyclic coordinate ascent algorithms (CCAA) decompose the original problem into a series of simple convex subproblems which can be solved efficiently. In particular, the transmit beamformers and receive beamformers are updated in an alternate manner, each update being a convex subproblem. Moreover, we show that both ECCAA and ICCAA are globally convergent to a stationary solution of the original problem. Finally, we present simulation results to illustrate the effectiveness of the proposed algorithms and compare them with the channel matched beamforming strategy and the leakage interference minimization solution.

Notation: We adopt the following notations in this paper. Lower and upper case letters in bold are used for vectors and matrices. For a given matrix $\mathbf{H}$, we denote its transpose, Hermitian, and inverse by $\mathbf{H}^T$, $\mathbf{H}^\dagger$, and $\mathbf{H}^{-1}$, respectively. Similarly, we denote the transpose and Hermitian of a vector $\mathbf{x}$ by $\mathbf{x}^T$ and $\mathbf{x}^\dagger$. We use $\|\mathbf{x}\|$ to represent the Euclidean norm of the vector $\mathbf{x}$. The notation $\mathbf{I}$ represents the identity matrix of an appropriate size. We use $\mathbf{e}_n^k$ to denote a n-dimensional column vector with its k-th element being one and other elements zero. The sets of real numbers and complex numbers are denoted by $\mathbb{R}$ and $\mathbb{C}$, respectively. Finally, we use $\mathcal{K} \triangleq \{1, 2, ..., K\}$ to denote the set of users.

II. Problem Formulation

Consider a K-user MIMO interference channel where the k-th transmitter and receiver are equipped with N_k and M_k antennas, respectively. By "user" in this paper we mean a transmitter-receiver pair (direct link). For the single-carrier channel, the received signal at receiver k is

$$\mathbf{y}_k = \mathbf{H}_{kk}\mathbf{v}_k s_k + \sum_{j \neq k} \mathbf{H}_{kj}\mathbf{v}_j s_j + \mathbf{z}_k,$$

where $\mathbf{H}_{kj} \in \mathbb{C}^{M_k \times N_j}$ is the channel matrix from transmitter j to receiver k, $\mathbf{v}_k \in \mathbb{C}^{N_k \times 1}$ is the beamformer used by transmitter k, $s_k \in \mathbb{C}$ is the symbol that transmitter k wishes to send to receiver k, and $\mathbf{z}_k \in \mathbb{C}^{M_k \times 1}$ is the additive white Gaussian noise (AWGN) with distribution $\mathcal{CN}(\mathbf{0}, \sigma_k^2 \mathbf{I})$. Each receiver uses a linear receive strategy and let $\mathbf{u}_k \in \mathbb{C}^{M_k \times 1}$ be the receive beamformer of receiver k. Then, the linearly processed signal at the k-th receiver is

$$\hat{s}_k = \mathbf{u}_k^\dagger \mathbf{y}_k.$$

Treating interference as noise, the SINR of user k can be written as

$$\text{SINR}_k = \frac{|\mathbf{u}_k^\dagger \mathbf{H}_{kk}\mathbf{v}_k|^2}{\sigma_k^2 \|\mathbf{u}_k\|^2 + \sum_{j \neq k} |\mathbf{u}_k^\dagger \mathbf{H}_{kj}\mathbf{v}_j|^2}.$$

The linear transceiver design problem is formulated as

$$\begin{aligned} \max_{\{\mathbf{u}, \mathbf{v}\}} \quad & U(\text{SINR}_1, \text{SINR}_2, ..., \text{SINR}_K) \\ \text{s.t.} \quad & \|\mathbf{u}_k\|^2 = 1, \ \|\mathbf{v}_k\|^2 \leq P_k, \ k \in \mathcal{K}, \end{aligned} \tag{1}$$

where P_k denotes the power budget of transmitter k, $U(\cdot)$ denotes the system utility, $\mathbf{u} = (\mathbf{u}_1; \mathbf{u}_2; ...; \mathbf{u}_K)$ and $\mathbf{v} = (\mathbf{v}_1; \mathbf{v}_2; ...; \mathbf{v}_K)$. A special case of (1) is the popular sum-rate maximization problem [8]–[11], where

$$U(\text{SINR}_1, \text{SINR}_2, ..., \text{SINR}_K) = \sum_{k \in \mathcal{K}} \log\left(1 + \text{SINR}_k\right).$$

Although the sum-rate maximization can achieve high network throughput, it sacrifices user fairness since only part of users with good channel states are served; the other users with poor channel conditions tend to be switched off and offered zero data rates. In this paper, we consider a more fair strategy which maximizes the minimum SINR (equivalent to the minimum rate) of all users in the system:

$$\begin{array}{ll} \max\limits_{\{\mathbf{u},\mathbf{v}\}} & G(\mathbf{u},\mathbf{v}) \triangleq \min\limits_{k\in\mathcal{K}} \{\text{SINR}_k\} \\ \text{s.t.} & \|\mathbf{u}_k\|^2 = 1,\ \|\mathbf{v}_k\|^2 \le P_k,\ k\in\mathcal{K}. \end{array} \tag{2}$$

Introducing an auxiliary variable

$$\text{SINR} = \min_{k\in\mathcal{K}} \{\text{SINR}_k\},$$

the minimum SINR maximization problem (2) can be rewritten as

$$\begin{array}{ll} \max\limits_{\{\mathbf{u},\mathbf{v}\}} & \text{SINR} \\ \text{s.t.} & \text{SINR} \le \text{SINR}_k,\ \|\mathbf{u}_k\|^2 = 1,\ \|\mathbf{v}_k\|^2 \le P_k,\ k\in\mathcal{K}. \end{array} \tag{3}$$

The max-min fairness linear transceiver design problem (2) and (3) are nonconvex due to the quadratic SINR constraints.

III. Computational Complexity Analysis

In practice, the number of antennas per user or base station is typically small, while the number of users in the system can be quite large. This motivates us to study the intrinsic complexity of the max-min fairness design problem (3) for increasing size of K, but with the number of antennas per user or base station fixed. As discussed in the introduction, for the single transmit/receive antenna case, this problem is polynomial time solvable. However, as we show in this section, when there are multiple antennas at each node, even the problem of checking the feasibility of a given set of target SINR levels is intrinsically intractable (strongly NP-hard in the sense of computational complexity theory [15]). This further implies that the optimization problem (2) and (3) are also intractable for large K (strongly NP-hard).

In computational complexity theory, a problem is said to be NP-hard if it is at least as hard as any problem in the class NP (problems that are solvable in Nondeterministic Polynomial time). The latter class includes such well known problems as checking if a given weighted graph has a tour that visits each node exactly once and whose tour length is no more than a given threshold (the traveling salesman problem). NP-complete problems are the hardest problems in NP in the sense that if any NP-complete problem is solvable in polynomial time, then each problem in NP is solvable in polynomial time. The traveling salesman is a NP-complete problem. NP-hard problems may not be in the class NP, but they are at least as hard as any NP-complete problem. It is widely believed that there can not exist a polynomial time algorithm to solve any NP-complete (or NP-hard) problem. Thus, once an optimization problem is shown to be NP-hard, we can no longer insist on having an efficient algorithm that can find its global optimum in polynomial time. Instead, we have to settle with less ambitious goals, such as finding high quality approximate solutions or locally optimal solutions of the problem in polynomial time.

The standard way to prove an optimization problem is NP-hard is to establish the NP-hardness of its corresponding decision problem. The latter is the problem to decide if the global minimum of the optimization problem is below a given threshold or not. The output of a decision problem is either true or false. The decision version of an optimization problem is usually in the class NP. Clearly, the decision version of an optimization problem is always easier than the optimization problem itself, since the latter further requires finding the global minimum value and the minimizer. Thus, if we show the decision version of an optimization problem is NP-hard, then the original optimization problem must also be NP-hard. For the max-min fairness design problem, the corresponding decision problem is simple to check if a given set of SINR levels can be satisfied by appropriate choices of linear transceivers.

In complexity theory, to show a decision problem $\mathcal{B}$ is NP-hard, we usually follow three steps: 1) choose a suitable known NP-complete decision problem $\mathcal{A}$; 2) construct a *polynomial time* transformation from any instance of $\mathcal{A}$ to an instance of $\mathcal{B}$; 3) prove under this transformation that any instance of problem $\mathcal{A}$ is true if and only if the instance of problem $\mathcal{B}$ is true.

We now establish the NP-hardness of the SINR feasibility problem: given a set of SINR levels and power budgets, does there exist a transmit/receive beamforming strategy for all users in the system so that each user's SINR level is greater than or equal to its given SINR target? To this end, we will transform an existing NP-complete problem, in our case the so-called 3SAT problem [15], into the SINR feasibility problem. The 3SAT problem is described as follows:

given m disjunctive clauses[1] defined on n Boolean variables such that each clause contains exactly three literals, the question is to check whether there exists a truth assignment for these Boolean variables such that all clauses are satisfied.

Theorem 3.1: Given a target minimum SINR $= \zeta$ in (3), the problem of checking the achievability of ζ is strongly NP-hard when each transmitter (receiver) is equipped with at least three antennas and each receiver (transmitter) is equipped with at least two antennas.

Proof: Without loss of generality, we consider the case $M_k = 3,\ N_k = 2,\ k \in \mathcal{K}$. The proof is based on a polynomial time transformation from the 3SAT problem. Specifically, we claim that the following feasibility problem is strongly NP-hard, i.e., checking whether there exist beamforming vectors $\mathbf{u}_k, \mathbf{v}_k\ (k \in \mathcal{K})$ such that all users' SINR levels are greater than or equal to the given SINR target ζ :

$$\begin{cases} \dfrac{|\mathbf{u}_k^\dagger \mathbf{H}_{kk} \mathbf{v}_k|^2}{\sigma_k^2 \|\mathbf{u}_k\|^2 + \sum\limits_{j \neq k} |\mathbf{u}_k^\dagger \mathbf{H}_{kj} \mathbf{v}_j|^2} \geq \zeta,\ k \in \mathcal{K}; \\ \|\mathbf{u}_k\|^2 = 1,\ k \in \mathcal{K}; \\ \|\mathbf{v}_k\|^2 \leq P_k,\ k \in \mathcal{K}. \end{cases}$$

In the above, the channel matrices are real-valued and $\mathbf{H}_{kj} \in \mathbb{R}^{3\times 2}, \forall\ k, j \in \mathcal{K}$. We remark that all the complexity results in this paper can extend to complex channel matrices.

Given any instance of the 3SAT problem consisting of m disjunctive clauses $c_1, c_2, ..., c_m$ defined on n Boolean variables $x_1, x_2, ..., x_n$, we construct below a MIMO interference channel with $m+n$ users, whereby the Boolean variable x_i corresponds to the i-th user, consisting of the "variable transmitter" i (V-TX$_i$) and the "variable receiver" i (V-RX$_i$); and the clause c_j corresponds to the $(n+j)$-th user, consisting of the "clause transmitter" j (C-TX$_j$) and the "clause receiver" j (C-RX$_j$). Hence, $\mathcal{K} = \{1, 2, ..., n+m\}$.

Now we construct the direct-link and crosstalk channel matrices $\mathbf{H}_{kj}$ among these $m+n$ users. To this end, we first define

$$\mathbf{H}_A = M\mathbf{e}_3^1 \left(\mathbf{e}_2^1\right)^T,\ \mathbf{H}_B = M\mathbf{e}_3^1 \left(\mathbf{e}_2^2\right)^T,\ \mathbf{H}_C = M\mathbf{e}_3^2 \left(\mathbf{e}_2^1\right)^T,$$

$$\mathbf{H}_D = M\mathbf{e}_3^2 \left(\mathbf{e}_2^2\right)^T,\ \mathbf{H}_E = M\mathbf{e}_3^3 \left(\mathbf{e}_2^1\right)^T,\ \mathbf{H}_F = M\mathbf{e}_3^3 \left(\mathbf{e}_2^2\right)^T,$$

[1]For a given set of Boolean variables, a literal is defined as either a Boolean variable x or its negation $\bar{x}$, while a disjunctive clause refers to a logical expression consisting of the logical "OR" of literals. For instance, $c = x \vee \bar{y} \vee z$ is a disjunctive clause over three literals $\{x,\ \bar{y},\ z\}$, and the clause c is satisfied provided that x is true, or y is false, or z is true.

where $M = 6\sqrt{2}$. All the direct-link channel matrices are set to be

$$\mathbf{H}_{kk} = \mathbf{E} \triangleq \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{pmatrix}, \; k \in \; \mathcal{K}.$$

The corresponding crosstalk channel matrices are: for user $k, \; k = 1, 2, ..., n$, let $\mathbf{H}_{kj} = \mathbf{0}, \; \forall \; j \in \mathcal{K} \backslash k$; and for user $k, \; k = n+1, n+2, ..., n+m$, $\mathbf{H}_{kj} = \mathbf{0}$ except

$$\begin{aligned} \mathbf{H}_{kj} &= \begin{cases} \mathbf{H}_B, & \text{if } \alpha_{\pi(k)} = x_j \text{ for some } j; \\ \mathbf{H}_A, & \text{if } \alpha_{\pi(k)} = \bar{x}_j \text{ for some } j, \end{cases} \\ \mathbf{H}_{kj} &= \begin{cases} \mathbf{H}_D, & \text{if } \beta_{\rho(k)} = x_j \text{ for some } j; \\ \mathbf{H}_C, & \text{if } \beta_{\rho(k)} = \bar{x}_j \text{ for some } j, \end{cases} \\ \mathbf{H}_{kj} &= \begin{cases} \mathbf{H}_F, & \text{if } \gamma_{\tau(k)} = x_j \text{ for some } j; \\ \mathbf{H}_E, & \text{if } \gamma_{\tau(k)} = \bar{x}_j \text{ for some } j, \end{cases} \end{aligned} \tag{4}$$

where $c_{k-n} = \alpha_{\pi(k)} \vee \beta_{\rho(k)} \vee \gamma_{\tau(k)}, \; \alpha, \; \beta$, and γ are taken from $\{x, \bar{x}\}$, and π, ρ, and τ are mappings from $\{n+1, n+2, ..., n+m\}$ to $\{1, 2, ..., n\}$.

To make the construction of channel matrices clear, an illustrative example is given, where there are three disjunctive clauses $c_1 = x_1 \vee \bar{x}_2 \vee x_3, \; c_2 = x_1 \vee x_2 \vee \bar{x}_4$, and $c_3 = \bar{x}_2 \vee \bar{x}_3 \vee x_4$ defined on four Boolean variables $\{x_1, x_2, x_3, x_4\}$. Then there are 7 users in the constructed MIMO interference network, including 4 "variable user" (denoted as user $1, 2, 3, 4$) and 3 "clause user" (denoted as user $5, 6, 7$). In this case, $\mathcal{K} = \{1, 2, ..., 7\}$. All direct-link channel matrices are $\mathbf{H}_{kk} = \mathbf{E}, \; \forall \; k \in \mathcal{K}$; while according to (4), all crosstalk channel matrices are zero except

$$\mathbf{H}_{5,1} = \mathbf{H}_B, \; \mathbf{H}_{5,2} = \mathbf{H}_C, \; \mathbf{H}_{5,3} = \mathbf{H}_F,$$

$$\mathbf{H}_{6,1} = \mathbf{H}_B, \; \mathbf{H}_{6,2} = \mathbf{H}_D, \; \mathbf{H}_{6,4} = \mathbf{H}_E,$$

$$\mathbf{H}_{7,2} = \mathbf{H}_A, \; \mathbf{H}_{7,3} = \mathbf{H}_C, \; \mathbf{H}_{7,4} = \mathbf{H}_F.$$

The constructed MIMO interference channel between V-TXs and C-RXs is depicted as Fig. 1.

Let $\sigma_k = 1, P_k = 1 \; (k \in \mathcal{K})$ and $\zeta = 1$. The constructed feasibility instance of problem (3) is

$$\begin{cases} \text{SINR}_k \geq 1, \; k \in \mathcal{K}; \\ \|\mathbf{u}_k\|^2 = 1, \; k \in \mathcal{K}; \\ \|\mathbf{v}_k\|^2 \leq 1, \; k \in \mathcal{K}, \end{cases} \tag{5}$$

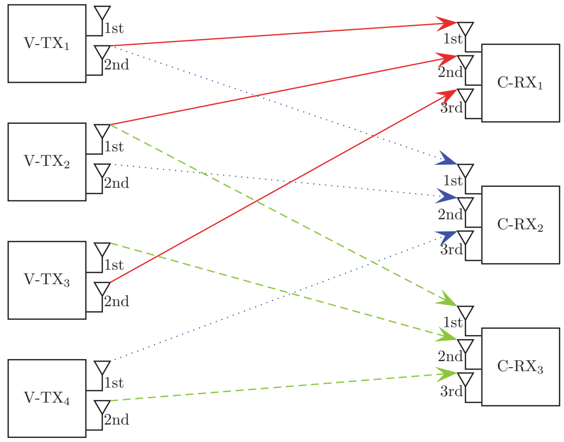

Fig. 1. An illustration of the constructed MIMO interference channel with $c_1 = x_1 \vee \bar{x}_2 \vee x_3,\ c_2 = x_1 \vee x_2 \vee \bar{x}_4,\ c_3 = \bar{x}_2 \vee \bar{x}_3 \vee x_4$, where all of V-TX$_1$, V-TX$_2$, V-TX$_3$, and V-TX$_4$ are equipped with two antennas while all of C-RX$_1$, C-RX$_2$, and C-RX$_3$ are equipped with three antennas. The arrows represent the channel matrices between the corresponding V-TXs and C-RXs. According to (4), the channel matrix between V-TX$_1$ and C-RX$_1$ is $\mathbf{H}_B$. When the transmitted signal vector at V-TX$_1$ is $\mathbf{v}_1 = (x_1, y_1)^T$, the received signal at C-RX$_1$ will be $My_1\mathbf{e}_3^1$, so we only connect the second antenna of V-TX$_1$ with the first antenna of C-RX$_1$. Similar connections apply to other V-TXs and C-RXs.

where

$$\text{SINR}_k = \begin{cases} |\mathbf{u}_k^\dagger \mathbf{E} \mathbf{v}_k|^2, & \text{if } k = 1, 2, ..., n; \\ \dfrac{|\mathbf{u}_k^\dagger \mathbf{E} \mathbf{v}_k|^2}{1 + \sum_{f \in \{\pi, \rho, \tau\}} |\mathbf{u}_k^\dagger \mathbf{H}_{kf(k)} \mathbf{v}_{f(k)}|^2}, & \text{otherwise.} \end{cases}$$

This is because the crosstalk channel matrices from all transmitters to V-RXs (receiver $i,\ i = 1, 2, ..., n$) are zero, so all of V-RXs are *interference-free*; while C-RXs (receiver $n + j,\ j = 1, 2, ..., m$) indeed suffers from crosstalk interferences from three V-TXs. Next, we show *the 3SAT problem is satisfied if and only if there exists a feasible solution to problem* (5).

Let $\mathbf{v}_k$ and $\mathbf{u}_k$ denote the beamforming vector associated with the k-th transmitter and receiver respectively. If there exists a truth assignment such that all the clauses in the 3SAT problem are satisfied, we claim that we can find beamforming vectors $\{\mathbf{u}_k, \mathbf{v}_k \mid k \in \mathcal{K}\}$ satisfying all the conditions in (5). In particular, we set

$$\mathbf{u}_k = \begin{cases} \mathbf{e}_3^1, & \text{if } k = 1, 2, ..., n; \\ \mathbf{e}_3^1,\ \mathbf{e}_3^2,\ \text{or } \mathbf{e}_3^3, & \text{if } k = n+1, n+2, ..., n+m, \end{cases}$$

and let $\mathbf{v}_k = (x_k, y_k)^T$ with

$$\begin{cases} y_k = 1 - x_k,\ x_k = 0 \text{ or } 1, & \text{if } k = 1, 2, ..., n; \\ x_k = 1,\ y_k = 0, & \text{if } k = n+1, n+2, ..., n+m. \end{cases} \tag{6}$$

In this way we define a group of beamforming vectors for (5), with $\mathbf{v}_k = \mathbf{e}_2^1$ or $\mathbf{e}_2^2$. We claim that, with these beamforming vectors, the interference subspace at each receiver has a dimension at most two and is contained in either span$\{\mathbf{e}_3^1, \mathbf{e}_3^2\}$, span$\{\mathbf{e}_3^1, \mathbf{e}_3^3\}$, or span$\{\mathbf{e}_3^2, \mathbf{e}_3^3\}$. As an illustrative example, suppose that $c_1 = x_1 \vee \bar{x}_2 \vee x_3$ is satisfied. Therefore, at least *one* of the following conditions $x_1 = 1$, $x_2 = 0$, or $x_3 = 1$ holds true. Consider the $(n+1)$-th receiver which corresponds to the clause c_1. The interference vectors at receiver $n+1$ are

$$\begin{aligned} \mathcal{I}_{n+1,1} &= \mathbf{H}_{n+1,1}\mathbf{v}_1 = \mathbf{H}_B\mathbf{v}_1 = My_1\mathbf{e}_3^1, \\ \mathcal{I}_{n+1,2} &= \mathbf{H}_{n+1,2}\mathbf{v}_2 = \mathbf{H}_C\mathbf{v}_2 = Mx_2\mathbf{e}_3^2, \\ \mathcal{I}_{n+1,3} &= \mathbf{H}_{n+1,3}\mathbf{v}_3 = \mathbf{H}_F\mathbf{v}_3 = My_3\mathbf{e}_3^3. \end{aligned} \tag{7}$$

Since at least one of the variables $y_1,\ x_2,\ y_3$ is zero, the interference subspace at receiver $n+1$ has dimension at most two, and is contained in either span$\{\mathbf{e}_3^1, \mathbf{e}_3^2\}$, span$\{\mathbf{e}_3^1, \mathbf{e}_3^3\}$, or span$\{\mathbf{e}_3^2, \mathbf{e}_3^3\}$. The same argument can be used for the other receivers $n+2, n+3, ..., n+m$. Moreover, $\mathbf{u}_k\,(k = n+1, n+2, ..., n+m)$ can be chosen from $\{\mathbf{e}_3^1, \mathbf{e}_3^2, \mathbf{e}_3^3\}$ such that it is orthogonal to the interference subspace at receiver k. The resulting leakage interference is then zero, i.e.,

$$\mathbf{u}_k^\dagger \mathbf{H}_{kj}\mathbf{v}_j = 0, \forall\ j \neq k,\ \ k = n+1, n+2, ..., n+m. \tag{8}$$

Let us check the SINR levels at all receivers:

- for user k, $k = 1, 2, ..., n$,

$$\text{SINR}_k = |\mathbf{u}_k^\dagger \mathbf{H}_{kk}\mathbf{v}_k|^2 = |\left(\mathbf{e}_3^1\right)^\dagger \mathbf{E}\mathbf{v}_k|^2 = 1$$

  due to $\mathbf{v}_k \in \{\mathbf{e}_2^1, \mathbf{e}_2^2\}$;

- for user k, $k = n+1, n+2, ..., n+m$,

$$\begin{aligned} \text{SINR}_k &= \frac{|\mathbf{u}_k^\dagger \mathbf{H}_{kk}\mathbf{v}_k|^2}{1 + \sum_{f \in \{\pi,\rho,\tau\}} |\mathbf{u}_k^\dagger \mathbf{H}_{kf(k)}\mathbf{v}_{f(k)}|^2} \\ &= |\mathbf{u}_k^\dagger \mathbf{H}_{kk}\mathbf{v}_k|^2 \quad \text{(from (8))} \\ &= |\mathbf{u}_k^\dagger \mathbf{E}\mathbf{e}_2^1|^2 \quad \text{(from (6))} \\ &= 1. \quad (\mathbf{u}_k \in \{\mathbf{e}_3^1, \mathbf{e}_3^2, \mathbf{e}_3^3\}) \end{aligned}$$

As a result, if there exists a truth assignment satisfying all the clauses in the given 3SAT problem, then we can construct a group of beamforming vector $\mathbf{u}_k, \mathbf{v}_k\,(k \in \mathcal{K})$ satisfying (5).

For the converse part, assuming that (5) has a feasible beamforming solution $\mathbf{u}_k, \mathbf{v}_k\,(k \in \mathcal{K})$, we claim that all the clauses in the given 3SAT problem can be satisfied. Denote $p_k = \|\mathbf{v}_k\|^2\,(k \in \mathcal{K})$. For user $k = 1, 2, ..., n$, we have

$$\begin{aligned} 1 &\leq \text{SINR}_k \\ &= |\mathbf{u}_k^\dagger \mathbf{E} \mathbf{v}_k|^2 \\ &\leq \max_{\|\mathbf{u}_k\|^2=1,\, \|\mathbf{v}_k\|^2=p_k} |\mathbf{u}_k^\dagger \mathbf{E} \mathbf{v}_k|^2 \\ &= \max_{\|\mathbf{v}_k\|^2=p_k} \lambda_{\max}\left(\mathbf{E}\mathbf{v}_k\mathbf{v}_k^\dagger\mathbf{E}^\dagger\right) \\ &= \max_{\|\mathbf{v}_k\|^2=p_k} \mathbf{v}_k^\dagger\mathbf{E}^\dagger\mathbf{E}\mathbf{v}_k \\ &= 6p_k, \end{aligned} \tag{9}$$

which gives

$$p_k \geq \frac{1}{6},\, k = 1, 2, ..., n. \tag{10}$$

From (9) and the fact that $\|\mathbf{v}_k\|^2 \leq 1$, we also have

$$|\mathbf{u}_k^\dagger \mathbf{H}_{kk} \mathbf{v}_k|^2 \leq 6,\, \forall\; k \in \mathcal{K}. \tag{11}$$

Let us define $\bar{\mathbf{v}}_k$ from $\mathbf{v}_k = (x_k, y_k)^T$ as follows:

$$\bar{\mathbf{v}}_k = \begin{cases} \mathbf{e}_2^1, & \text{if } |x_k| \geq |y_k|; \\ \mathbf{e}_2^2, & \text{if } |x_k| < |y_k|. \end{cases} \tag{12}$$

We prove by contradiction that the beamforming vectors (12) can make the dimension of the interference subspace at each receiver $k\;(k = n+1, n+2, ..., n+m)$ at most two. To see why this is the case, consider the previous example and assume that at receiver $n+1$ the dimension of the interference subspace is three. Since $\bar{\mathbf{v}}_k = \mathbf{e}_2^1$ or $\mathbf{e}_2^2$ (c.f. (12)) and there are three potential interferers to receiver $n+1$, it follows from (7) that each of the interferer can cause either zero interference or an interference along the direction $\mathbf{e}_3^i$, $i = 1, 2, 3$. Thus, for the interference subspace at receiver $n+1$ to have dimension 3, the interference subspace at receiver $n+1$ must be spanned by

$$\bar{\mathcal{I}}_{n+1,1} = M\mathbf{e}_3^1,\; \bar{\mathcal{I}}_{n+1,2} = M\mathbf{e}_3^2,\; \bar{\mathcal{I}}_{n+1,3} = M\mathbf{e}_3^3. \tag{13}$$

Combining (7), (12), and (13), we have

$$|y_1| \geq |x_1|,\; |x_2| \geq |y_2|,\; |y_3| \geq |x_3|. \tag{14}$$

Let

$$p = \min_{1 \le k \le n} \{p_k\}. \tag{15}$$

Consider the SINR at the $(n+1)$-th receiver,

$$\begin{aligned}
&\text{SINR}_{n+1} \\
&= \frac{|\mathbf{u}_{n+1}^{\dagger}\mathbf{H}_{n+1,n+1}\mathbf{v}_{n+1}|^2}{1 + |\mathbf{u}_{n+1}^{\dagger}\mathbf{H}_B\mathbf{v}_1|^2 + |\mathbf{u}_{n+1}^{\dagger}\mathbf{H}_C\mathbf{v}_2|^2 + |\mathbf{u}_{n+1}^{\dagger}\mathbf{H}_F\mathbf{v}_3|^2} \\
&\le \frac{6}{1 + |\mathbf{u}_{n+1}^{\dagger}\mathbf{H}_B\mathbf{v}_1|^2 + |\mathbf{u}_{n+1}^{\dagger}\mathbf{H}_C\mathbf{v}_2|^2 + |\mathbf{u}_{n+1}^{\dagger}\mathbf{H}_F\mathbf{v}_3|^2} \\
&= \frac{6}{1 + |\mathbf{u}_{n+1}^{\dagger}\mathcal{I}_{n+1,1}|^2 + |\mathbf{u}_{n+1}^{\dagger}\mathcal{I}_{n+1,2}|^2 + |\mathbf{u}_{n+1}^{\dagger}\mathcal{I}_{n+1,3}|^2} \\
&\le \frac{6}{1 + M^2\frac{p}{2}\left(|\mathbf{u}_{n+1}^{\dagger}\mathbf{e}_3^1|^2 + |\mathbf{u}_{n+1}^{\dagger}\mathbf{e}_3^2|^2 + |\mathbf{u}_{n+1}^{\dagger}\mathbf{e}_3^3|^2\right)} \\
&= \frac{6}{1 + M^2 p/2} \quad (\|\mathbf{u}_{n+1}\|^2 = 1) \\
&< 1, (\text{from } M^2 = 72,\ (10),\ \text{and } (15))
\end{aligned}$$

where the first inequality is due to (11) and the second inequality is due to (7) and (14). This contradicts the fact $\text{SINR}_k \ge 1\,(k \in \mathcal{K})$. As a result, we can choose $\bar{\mathbf{u}}_k \in \{\mathbf{e}_3^1, \mathbf{e}_3^2, \mathbf{e}_3^3\}$ such that

$$\bar{\mathbf{u}}_k^{\dagger}\mathbf{H}_{kj}\bar{\mathbf{v}}_j = 0, \forall\ j \ne k.$$

Actually, (12) gives a truth assignment which satisfies all the clauses in the 3SAT problem as in [16].

Finally, this transformation is in polynomial time. Since the 3SAT problem is NP-complete, we can conclude that the problem of checking whether the given target SINR is feasible is strongly NP-hard. ■

The max-min fairness linear transceiver design problem is shown to remain NP-hard for the scenario where all transmitters and receivers are equipped with exactly two antennas ($M_k = N_k = 2, k \in \mathcal{K}$) in a late paper [17]. Their proof is also based on a polynomial time transformation from the same NP-hard problem as the one used in this paper. We point out that the NP-hardness result in [17] does not imply our result, since the result in [17] holds true only for complex channel matrices, while our result holds true for both complex and real channel matrices.

IV. Cyclic Coordinate Ascent Algorithm

We develop cyclic coordinate ascent algorithms (CCAA) for the max-min fairness linear transceiver design problem (2). The basic idea of CCAA is to partition the design variables into different blocks and cyclicly solve the problem with respect to one block while fixing the others. To exploit the separable structure of the constraints, we partition the variables in problem (2) into two blocks $\mathbf{u}=(\mathbf{u}_1;...;\mathbf{u}_K)$ and $\mathbf{v}=(\mathbf{v}_1;...;\mathbf{v}_K)$. Denote by n the iteration index and assume that $\mathbf{v}^0$ is given. By fixing $\mathbf{v}=\mathbf{v}^n$ $(n\geq 0)$, problem (2) with respect to $\mathbf{u}$ can be solved by solving K independent small problems

$$\begin{aligned} \max_{\{\mathbf{u}_k\}} \quad & \frac{|\mathbf{u}_k^\dagger\mathbf{H}_{kk}\mathbf{v}_k^n|^2}{\sigma_k^2\|\mathbf{u}_k\|^2+\sum\limits_{j\neq k}|\mathbf{u}_k^\dagger\mathbf{H}_{kj}\mathbf{v}_j^n|^2} \\ \text{s.t.} \quad & \|\mathbf{u}_k\|^2=1, \end{aligned} \tag{16}$$

for $k=1,\ldots,K$. Defining

$$\mathbf{M}_k(\mathbf{v})=\left(\sum_{j\in\mathcal{K}}\mathbf{H}_{kj}\mathbf{v}_j\left(\mathbf{H}_{kj}\mathbf{v}_j\right)^\dagger+\sigma_k^2\mathbf{I}\right)^{-1},\ \forall\ k\in\mathcal{K},$$

the optimal solution $\mathbf{u}_k^n$ to problem (16) is the linear minimum mean square error (LMMSE) receive beamformer

$$\mathbf{u}_k^n=\tilde{\mathbf{u}}_k^n/\|\tilde{\mathbf{u}}_k^n\|,\ \tilde{\mathbf{u}}_k^n=\mathbf{M}_k(\mathbf{v}^n)\mathbf{H}_{kk}\mathbf{v}_k^n. \tag{17}$$

Then fixing $\mathbf{u}=\mathbf{u}^n$, we solve problem (2) with regard to $\mathbf{v}$ for $\mathbf{v}^{n+1}$. In this case, the problem is

$$\begin{aligned} \max_{\{\mathbf{v}\}} \quad & \min_{k\in\mathcal{K}}\left\{\frac{|\left(\mathbf{u}_k^n\right)^\dagger\mathbf{H}_{kk}\mathbf{v}_k|^2}{\sigma_k^2+\sum_{j\neq k}|\left(\mathbf{u}_k^n\right)^\dagger\mathbf{H}_{kj}\mathbf{v}_j|^2}\right\} \\ \text{s.t.} \quad & \|\mathbf{v}_k\|^2\leq P_k,\ k\in\mathcal{K}. \end{aligned} \tag{18}$$

We call the corresponding algorithm by exact (inexact) cyclic coordinate ascent algorithm if (18) is solved exactly (inexactly) for the transmit beamformer $\mathbf{v}^{n+1}$. They are abbreviated by ECCAA and ICCAA, respectively.

A. Exact Cyclic Coordinate Ascent Algorithm

Consider the ECCAA for problem (2). For convenience, define the mapping $\mathbf{u}=\phi(\mathbf{v})$ with its k-th block $\mathbf{u}_k=\phi_k(\mathbf{v})=\tilde{\mathbf{u}}_k/\|\tilde{\mathbf{u}}_k\|,\tilde{\mathbf{u}}_k=\mathbf{M}_k(\mathbf{v})\mathbf{H}_{kk}\mathbf{v}_k,$ and $\Phi(\mathbf{v}^n)$ to be the optimal solution set of problem (2) with $\mathbf{v}=\mathbf{v}^n$ fixed (the optimal LMMSE solution is not unique, as any optimal solution multiplied by a complex unit norm scalar will remain optimal). Noticing that $\phi_k(\mathbf{v}^n)$ is exactly the vector in (17), so $\mathbf{u}^n = \phi(\mathbf{v}^n) \in \Phi(\mathbf{v}^n)$.

In ECCAA, when $\mathbf{u}^n$ is fixed, we solve problem (18) exactly for an optimal transmit beamformer $\mathbf{v}^{n+1} \in \Psi(\mathbf{u}^n)$, where $\Psi(\mathbf{u}^n)$ represents the optimal solution set of problem (18) (the solution of problem (18) is not unique in general due to the quasi-convexity of the problem with respect to $\mathbf{v}$ [4]). By [10], problem (18) can be solved to global optimality in polynomial time using a bisection procedure, where each step solves a second order cone programming (SOCP) [18].

Denote $G_{2n} = G(\mathbf{u}^n, \mathbf{v}^n)$ and $G_{2n+1} = G(\mathbf{u}^n, \mathbf{v}^{n+1})$, where G is the objective function in problem (2). The following stopping criterion is used to terminate the algorithm,

$$\frac{G_{2n+1} - G_{2n-1}}{\max\{G_{2n-1}, 1\}} \le \epsilon, \tag{19}$$

where ϵ is the prescribed stopping tolerance. A detailed description of ECCAA is given as follows.

ECCAA

S1. Initialization: Given $\mathbf{v}^0$ and tolerance ϵ. Set $n = 0$.

S2. Computing $\mathbf{u}^n$: Compute the optimal LMMSE receive beamformer $\mathbf{u}^n = \phi(\mathbf{v}^n) \in \Phi(\mathbf{v}^n)$ with $\mathbf{v} = \mathbf{v}^n$ fixed.

S3. Computing $\mathbf{v}^{n+1}$: Solve problem (18) to obtain the optimal transmit beamformer $\mathbf{v}^{n+1} \in \Psi(\mathbf{u}^n)$ with $\mathbf{u} = \mathbf{u}^n$ fixed.

S4. Termination:
- if (19) is satisfied, terminate the algorithm;
- else set $n = n + 1$ and go to **S2**.

In general, cyclic coordinate algorithms may not converge to a KKT solution even if each subproblem is exactly solved [19]. It is known from [20] that the separability of the constraints is a necessary condition for the convergence. The book [21] established some convergence results for cyclic coordinate algorithms under strong assumptions. For example, the objective function is required to be *continuously differentiable* over the feasible set and the minimizer (maximizer) in terms of each block is assumed to be *unique*. Noticing that the objective function in problem (2) is non-differentiable and that the optimal solution of (16) is not unique, the result of [21] is not applicable to the ECCAA. In fact, the algorithm similar as the ECCAA has already been proposed to solve the linear transceiver design problem in different scenarios, i.e., the problem of minimizing the total transmission power subject to quality of service constraints in [22] and the problem of maximizing the minimum SINR for the MIMO downlink channel in [12], but the convergence of the algorithm is not guaranteed. Nevertheless, the next result shows the global convergence of ECCAA for the max-min fairness linear transceiver design problem (2).

Theorem 4.1: Consider the ECCAA for solving problem (2) with $\epsilon = 0$. Then the generated sequence $\{(\mathbf{u}^n, \mathbf{v}^n)\}$ either terminates at a stationary point[2] finitely or every accumulation point of the sequence is a stationary point of (2).

Proof: Suppose that the sequence $\{(\mathbf{u}^n, \mathbf{v}^n)\}$ does not terminate finitely. Since both $\{\|\mathbf{u}^n\|\}$ and $\{\|\mathbf{v}^n\|\}$ are bounded, they have convergent subsequences. Assume without loss of generality that $\mathbf{u}^n \to \bar{\mathbf{u}}$ and $\mathbf{v}^n \to \bar{\mathbf{v}}$ for $n \in \mathcal{N}$, where $\mathcal{N}$ is some infinite subsequence. Consequently, we have

$$G_{2n} = G(\mathbf{u}^n, \mathbf{v}^n) \to G(\bar{\mathbf{u}}, \bar{\mathbf{v}}) \triangleq \bar{G},\ n \in \mathcal{N}.$$

By the ECCAA, the sequence $\{G_n\}$ is nondecreasing and hence $G_n \to \bar{G}$.

The outline of the remaining proof is as follows. We first claim that $\bar{\mathbf{u}} \in \Phi(\bar{\mathbf{v}})$ and $\bar{\mathbf{v}} \in \Psi(\bar{\mathbf{u}})$, which imply that $\bar{\mathbf{u}}$ is an optimal solution of problem (2) with $\mathbf{v} = \bar{\mathbf{v}}$ fixed, and $\bar{\mathbf{v}}$ is an optimal solution of problem (2) with $\mathbf{u} = \bar{\mathbf{u}}$ fixed, respectively. Then we combine these two facts to prove that $(\bar{\mathbf{u}}, \bar{\mathbf{v}})$ is a stationary point of problem (2).

From the fact that $\text{SINR}_k(\mathbf{u}, \mathbf{v})$ only depends on $\mathbf{u}_k$ with fixed $\mathbf{v}$ and the optimality of $\mathbf{u}^n$, we have for any feasible $\mathbf{u}$ that

$$\text{SINR}_k(\mathbf{u}^n, \mathbf{v}^n) \geq \text{SINR}_k(\mathbf{u}, \mathbf{v}^n),\ \forall\ k \in \mathcal{K}.$$

Taking limits from both sides of the above inequality, it follows that

$$\text{SINR}_k(\bar{\mathbf{u}}, \bar{\mathbf{v}}) \geq \text{SINR}_k(\mathbf{u}, \bar{\mathbf{v}}),\ \forall\ k \in \mathcal{K},$$

[2]A point $\mathbf{x}$ is said to be a stationary point (KKT point) of a maximization problem $\mathcal{P}$ if it satisfies the KKT condition of problem $\mathcal{P}$. Under some mild conditions [23], the stationarity of $\mathbf{x}$ implies its local optimality in the sense that there is no feasible ascent direction at point $\mathbf{x}$ for problem $\mathcal{P}$. However, the stationarity of $\mathbf{x}$ does not imply its global optimality in general (unless problem $\mathcal{P}$ is convex).

which implies that $\bar{\mathbf{u}} \in \Phi(\bar{\mathbf{v}})$. Moreover, because of $\mathbf{v}^{n+1} \in \Psi(\mathbf{u}^n)$, for any feasible $\mathbf{v}$, we have

$$G(\mathbf{u}^n, \mathbf{v}) \le G(\mathbf{u}^n, \mathbf{v}^{n+1}) \le G(\bar{\mathbf{u}}, \bar{\mathbf{v}}),\ n \in \mathcal{N}.$$

Letting n go to infinity in the above inequality, we obtain $G(\bar{\mathbf{u}}, \mathbf{v}) \le G(\bar{\mathbf{u}}, \bar{\mathbf{v}})$. Thus, $\bar{\mathbf{v}} \in \Psi(\bar{\mathbf{u}})$.

Now we prove that $(\bar{\mathbf{u}}, \bar{\mathbf{v}})$ is a stationary point of problem (2). It is clear that $(\bar{\mathbf{u}}, \bar{\mathbf{v}})$ is feasible. On one hand, the fact $\bar{\mathbf{u}} \in \Phi(\bar{\mathbf{v}})$ (i.e., $\bar{\mathbf{u}}_k$ solves problem (16) with $\mathbf{v}^n$ there replaced with $\bar{\mathbf{v}}$.) implies that there exist multipliers $\{\eta_k\}_{k\in\mathcal{K}}$ satisfying

$$\nabla_{\mathbf{u}_k}\mathrm{SINR}_k(\bar{\mathbf{u}}, \bar{\mathbf{v}}) = 2\eta_k\bar{\mathbf{u}}_k,\ \forall\ k \in \mathcal{K}. \tag{20}$$

On the other hand, we know from $\bar{\mathbf{v}} \in \Psi(\bar{\mathbf{u}})$ (i.e., $\bar{\mathbf{v}}$ solves problem (18) with $\mathbf{u}^n$ there replaced with $\bar{\mathbf{u}}$.) that there exist multipliers $\{\lambda_k\}_{k\in\mathcal{K}}$ and $\{\mu_k\}_{k\in\mathcal{K}}$ such that

$$\begin{cases} \sum_{k\in\mathcal{K}} \lambda_k \nabla_{\mathbf{v}_i}\mathrm{SINR}_k(\bar{\mathbf{u}}, \bar{\mathbf{v}}) = 2\mu_i\bar{\mathbf{v}}_i, \forall\ i \in \mathcal{K}, \\ \sum_{k\in\mathcal{K}} \lambda_k = 1,\ \lambda_k = 0 \text{ if } \mathrm{SINR}_k(\bar{\mathbf{u}}, \bar{\mathbf{v}}) > \bar{G}, \\ \lambda_k \ge 0,\ \mu_k \ge 0,\ \mu_k(P_k - \|\bar{\mathbf{v}}_k\|^2) = 0, \forall\ k \in \mathcal{K}. \end{cases} \tag{21}$$

Multiplying (20) by λ_k and combining it with (21) and the feasibility condition of $(\bar{\mathbf{u}}, \bar{\mathbf{v}})$, we obtain the KKT conditions of problem (2) as follows:

$$\begin{cases} \sum\limits_{k\in\mathcal{K}} \lambda_k \nabla_{\mathbf{v}_i}\mathrm{SINR}_k(\bar{\mathbf{u}}, \bar{\mathbf{v}}) = 2\mu_i\bar{\mathbf{v}}_i, \forall\ i \in \mathcal{K}, \\ \sum\limits_{k\in\mathcal{K}} \lambda_k \nabla_{\mathbf{u}_i}\mathrm{SINR}_k(\bar{\mathbf{u}}, \bar{\mathbf{v}}) = 2\tau_i\bar{\mathbf{u}}_i, \forall\ i \in \mathcal{K}, \\ \sum\limits_{k\in\mathcal{K}} \lambda_k = 1,\ \lambda_k = 0 \text{ if } \mathrm{SINR}_k(\bar{\mathbf{u}}, \bar{\mathbf{v}}) > \bar{G}, \\ \|\bar{\mathbf{u}}_k\|^2 = 1,\ \|\bar{\mathbf{v}}_k\|^2 \le P_k, \forall\ k \in \mathcal{K}, \\ \lambda_k \ge 0,\ \mu_k \ge 0,\ \mu_k(P_k - \|\bar{\mathbf{v}}_k\|^2) = 0, \forall\ k \in \mathcal{K}, \end{cases}$$

where $\mu_k \ge 0$ and $\tau_k = \lambda_k\eta_k$ serve as the Lagrangian multipliers corresponding to the constraints $P_k - \|\mathbf{v}_k\|^2 \ge 0$ and $\|\mathbf{u}_k\|^2 - 1 = 0$, respectively. Hence $(\bar{\mathbf{u}}, \bar{\mathbf{v}})$ is a stationary point of problem (2). ■

B. Inexact Cyclic Coordinate Ascent Algorithm

In ECCAA, the exact solution of problem (18) for a new transmit beamformer $\mathbf{v}^{n+1}$ requires solving a sequence of SOCP feasibility problems [10] and hence is computationally expensive. It would be interesting to design a more practical scheme for updating $\mathbf{v}^{n+1}$ that not only has a moderate computation cost but also preserves the monotonicity and the global convergence of ECCAA. By this motivation, we consider an inexact cyclic coordinate ascent algorithm, in which the transmit beamformer $\mathbf{v}^{n+1}$ is the solution of the problem

$$\begin{aligned} \max_{\{\mathbf{v},\theta\}} \quad & \theta \\ \text{s.t.} \quad & \frac{(\mathbf{u}_k^n)^\dagger \mathbf{H}_{kk}\mathbf{v}_k - \theta}{\sqrt{\sigma_k^2 + \sum_{j\neq k} |(\mathbf{u}_k^n)^\dagger \mathbf{H}_{kj}\mathbf{v}_j|^2}} \geq \sqrt{G_{2n}},\ k \in \mathcal{K}, \\ & \|\mathbf{v}_k\|^2 \leq P_k,\ k \in \mathcal{K}, \end{aligned} \tag{22}$$

where $G_{2n} = G(\mathbf{u}^n, \mathbf{v}^n)$ as before. It is worth pointing out that if we solve problem (22) (assuming the solution of problem (22) is $\hat{\mathbf{v}}^n$) and update $G_{2n} = G(\mathbf{u}^n, \hat{\mathbf{v}}^n)$ in it iteratively, we can solve the transmit beamforming design problem (18) with fixed receiver $\mathbf{u} = \mathbf{u}^n$ to global optimality [24]. However, here we just solve problem (22) *once* to reduce the computational cost of updating the transmit beamforming vector. For notational simplicity, we define $D_k(\mathbf{u},\mathbf{v})$ and $I_k(\mathbf{u},\mathbf{v})$ as

$$D_k(\mathbf{u},\mathbf{v}) = \mathbf{u}_k^\dagger \mathbf{H}_{kk}\mathbf{v}_k,\ I_k(\mathbf{u},\mathbf{v}) = \sqrt{\sigma_k^2 + \sum_{j\neq k} |\mathbf{u}_k^\dagger \mathbf{H}_{kj}\mathbf{v}_j|^2},\ \forall\ k \in \mathcal{K}.$$

It follows from the optimality of $\mathbf{v}^{n+1}$ that $D_k(\mathbf{u}^n, \mathbf{v}^{n+1}) > 0,\ \forall\ k \in \mathcal{K}$.

The above problem (22) is feasible since we can appropriately rotate $\mathbf{v}_k^n$ $(k \in \mathcal{K})$ so that $(\mathbf{u}_k^n)^\dagger \mathbf{H}_{kk}\mathbf{v}_k^n > 0$ $(\forall\ k \in \mathcal{K})$ and hence $(\mathbf{v}^n, 0)$ is a feasible point for problem (22). Consequently, the optimal value θ^{n+1} of (22) is always nonnegative and $G_{2n+1} \geq G_{2n}$. This, with $G_{2n} \geq G_{2n-1}$ (due to the choice of $\mathbf{u}^n$), implies that the minimum SINR sequence $\{G_n\}$ is monotonically increasing. Further, since the optimal solution $(\mathbf{v}^{n+1}, \theta^{n+1})$ must be such that at least one of the inequality constraints involving θ is met with equality, we can express the optimal value θ^{n+1} of problem (22) as

$$\theta^{n+1} = \min_{k\in\mathcal{K}} \left\{ D_k(\mathbf{u}^n, \mathbf{v}^{n+1}) - \sqrt{G_{2n}} I_k(\mathbf{u}^n, \mathbf{v}^{n+1}) \right\}. \tag{23}$$

As the solution to problem (22) does not solve problem (18) in general, we denote the algorithm corresponding to problem (22) for the transmit beamformer $\mathbf{v}^{n+1}$ by ICCAA. The whole description of ICCAA is given as follows.

ICCAA

S1. Initialization: Given $\mathbf{v}^0$ and tolerance ϵ. Set $n = 0$.

S2. Computing $\mathbf{u}^n$: Compute the optimal LMMSE receive beamformer $\mathbf{u}^n = \phi(\mathbf{v}^n) \in \Phi(\mathbf{v}^n)$ with $\mathbf{v} = \mathbf{v}^n$ fixed.

S3. Computing $\mathbf{v}^{n+1}$: Solve problem (22) to obtain the transmit beamformer $\mathbf{v}^{n+1}$ with $\mathbf{u} = \mathbf{u}^n$ fixed.

S4. Termination:
- if (19) is satisfied, terminate the algorithm;
- else set $n = n + 1$ and go to **S2**.

Notice that problem (22) is an SOCP and hence can be solved to global optimality in polynomial time. In contrast, to update the transmit beamformer from $\mathbf{v}^n$ to $\mathbf{v}^{n+1}$, the ECCAA requires solving a sequence of SOCP feasibility problems in the binary search step. Hence, the ICCAA, which only requires solving one SOCP, significantly reduces the computation cost of $\mathbf{v}^{n+1}$ per iteration. Although we update the transmit beamforming vector in an inexact manner in ICCAA, we still have the following global convergence result for it. See Appendix A for the proof.

Theorem 4.2: Consider the ICCAA for solving problem (2) with $\epsilon = 0$. Then either the generated sequence $\{(\mathbf{u}^n, \mathbf{v}^n)\}$ terminates at a stationary point or any of its accumulation point $(\bar{\mathbf{u}}, \bar{\mathbf{v}})$ is a stationary point.

V. NUMERICAL RESULTS

In this section, we present some numerical simulations to evaluate the effectiveness of the proposed ECCAA and ICCAA. We consider a MIMO multi-user interference channel with three antennas ($N_k = 3$) at each transmitter and two antennas ($M_k = 2$) at each receiver. In a similar way as in [8] and [11], the channel matrices are generated according to the complex Gaussian distribution $\text{vec}(\mathbf{H}_{kj}) \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}), \forall\ k, j \in \mathcal{K}$. The transmit power budget and the noise power are set to 1 and σ^2 for all users. Define SNR$= -10\log_{10}(\sigma^2)$. The following suboptimal solutions are adopted as benchmarks:

1. Benchmark1 is the so-called channel matched beamformer, which solves the following

maximization problem

$$(\tilde{\mathbf{u}}_k, \tilde{\mathbf{v}}_k) = \arg \max_{\{\mathbf{u}_k, \mathbf{v}_k\}} \quad |\mathbf{u}_k^\dagger \mathbf{H}_{kk} \mathbf{v}_k|^2 \quad \text{s.t.} \quad \|\mathbf{u}_k\|^2 = 1,\ \|\mathbf{v}_k\|^2 \leq P_k.$$

Notice $\tilde{\mathbf{u}}_k$ and $\tilde{\mathbf{v}}_k$ are the scaled left and right singular vectors corresponding to the largest singular value of the direct-link channel matrix $\mathbf{H}_{kk}$. In this case, each user selfishly maximizes its own received signal power. The channel matched beamformer scheme is optimal when the network has only one user.

2. Benchmark2 is the leakage interference minimization solution. Leakage interference minimization problem

$$\begin{aligned} \min_{\{\mathbf{u}, \mathbf{v}\}} \quad & \sum_{k \in \mathcal{K}} \sum_{j \neq k} |\mathbf{u}_k^\dagger \mathbf{H}_{kj} \mathbf{v}_j|^2 \\ \text{s.t.} \quad & \|\mathbf{u}_k\|^2 = 1,\ \|\mathbf{v}_k\|^2 \leq P_k,\ k \in \mathcal{K} \end{aligned} \tag{24}$$

is proposed in [25] as an effective method of checking the feasibility of interference alignment [26] and finding the solution of achieving interference alignment (if there exists such a solution). However, problem (24) is shown to be strongly NP-hard in [16] when each node is equipped with more than one antenna. We use the proposed algorithm in [25] to obtain one suboptimal solution of problem (24).

An upper bound on the optimal value of linear transceiver design problem (2) is given by

$$\min_{k \in \mathcal{K}} \left\{ |\tilde{\mathbf{u}}_k^\dagger \mathbf{H}_{kk} \tilde{\mathbf{v}}_k|^2 / \sigma_k^2 \right\},$$

which is not achievable in general. However, it can still be used as an ultimate upper bound. In our simulations, both the ICCAA and the ECCAA are initialized to the channel matched beamformer (Benchmark1), and the stopping criterion parameter ϵ in (19) is set to be 10^{-3}. We use CVX [27] to solve the related SOCP problems. All figures are obtained by averaging over 200 independent channel realizations except Fig. 2.

Fig. 2 plots the convergence behavior of the proposed ICCAA and ECCAA for the case of $K = 5$ and SNR $= 15$ dB. It can be seen that the min-SINR values generated by the ICCAA and the ECCAA indeed increase monotonically as expected, and that the most of improvement is achieved in the first few iterations. These two properties make the proposed algorithms attractive for practical implementations. In general, one update of the transmit beamformer in ECCAA yields more gain than in ICCAA. This is because the transmit beamformer is exactly solved

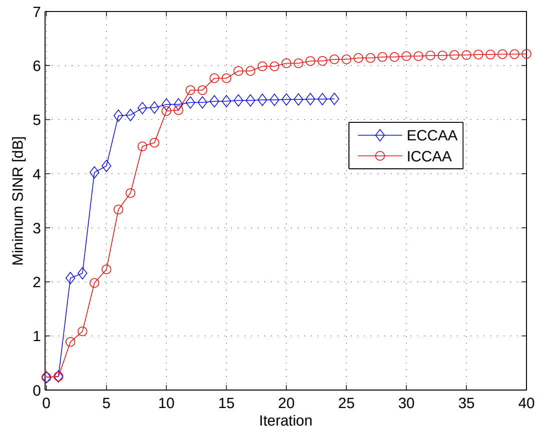

Fig. 2. A convergence example of ICCAA and ECCAA with $K = 5$ and SNR $= 15$ dB.

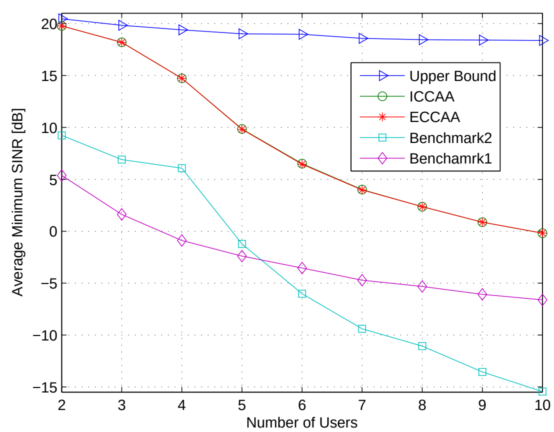

Fig. 3. Average minimum SINR versus the number of users with SNR $= 15$ dB.

in ECCAA (but inexactly solved in the ICCAA). Compared to the ECCAA, the ICCAA takes more iterations to converge, but needs far less computation cost per iteration. We can also see from Fig. 2 that the performance of ICCAA is better than that of the ECCAA. This is due to the fact that the algorithms may converge to different KKT solutions.

Fig. 3 shows the average minimum SINR performance comparison of the proposed algorithms and benchmarks versus different number of users, while Fig. 4 depicts the the average minimum SINR performance vs. the SNR. Figs. 3 and 4 show that the proposed algorithms significantly outperform the benchmarks in terms of the achieved minimum SINR value, and achieve almost

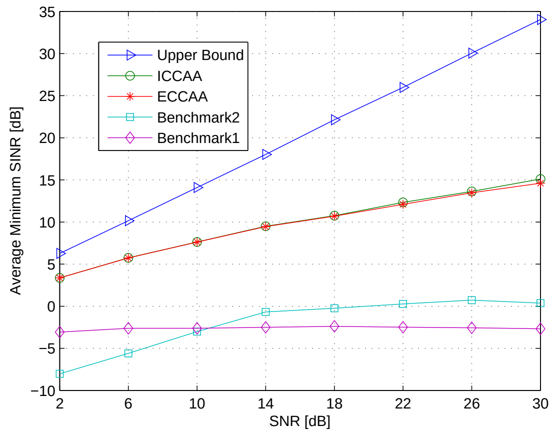

Fig. 4. Average minimum SINR versus SNR with $K = 5$.

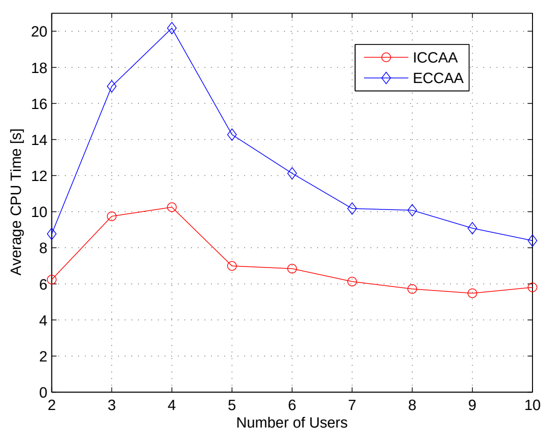

Fig. 5. Average CPU time versus the number of users with SNR $= 15$ dB.

50% of the interference-free max-min SINR value when the number of users in the network is small (e.g., $K \leq 5$). One can also observe from Figs. 3 and 4 that the ICCAA and the ECCAA yield almost the same performance. Actually, the performance of ICCAA is slightly better than the one of ECCAA in our simulations. This difference becomes obvious in Fig. 4 for large SNR values. We also applied the ICCAA to solve the minimum SINR maximization problem for the SIMO interference channel in [13], and we found that the ICCAA always solves the problem to global optimality.

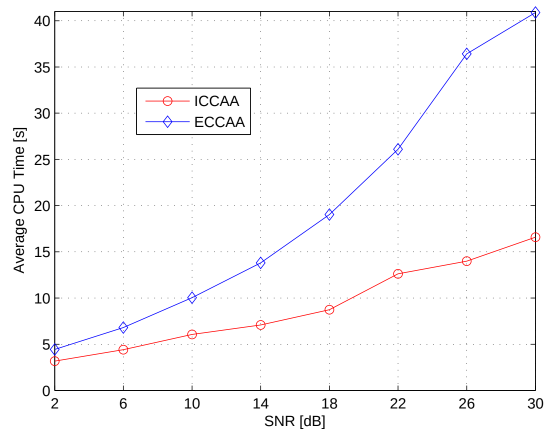

Fig. 6. Average CPU time versus SNR with $K = 5$.

The average CPU time comparison of the ICCAA and the ECCAA versus different number of users and different SNRs are illustrated as Fig. 5 and Fig. 6, respectively. We can observe that the ICCAA substantially outperforms the ECCAA in terms of the average CPU time. This is not surprising since in each update of the transmit beamformer, a sequence of SOCPs need to be solved in the ECCAA while only one SOCP is solved in the ICCAA. One may say that it is not intuitive that the CPU time decreases as the number of users increases in Fig. 5. The reason why this "strange" phenomenon happens is because we initialize the two iterative algorithms[3], ECCAA and ICCAA, to the channel matched beamformer. We know that the channel matched beamformer is nearly optimal in the strong interference channel. As the number of users in the network increases, the interference level in the network becomes higher. Thus, the channel matched beamformer becomes closer to the true solution, and it takes less CPU time for the two algorithms to terminate.

VI. Conclusion

In this paper, we consider the max-min fairness linear transceiver design problem for a multi-user MIMO interference channel. A major design challenge is to find the globally optimal transceiver to maximize the minimum SINR among all users. We first show in this paper that,

[3]How long it takes an iterative algorithm to terminate of course depends on the initial point.

when each transmitter (receiver) is equipped with more than a single antenna and each receiver (transmitter) is equipped with more than two antennas, the max-min fairness linear transceiver design problem is computationally intractable (strongly NP-hard) as the number of users in the system increases. Motivated by the complexity result, we then propose two iterative algorithms based on the cyclic coordinate ascent strategy, ECCAA and ICCAA, for the max-min fairness linear transceiver design problem. The proposed algorithms alternately optimize the transmit beamformer (including the power allocation) and the receive beamformer, and thus decompose the original NP-hard problem into a series of easily solvable convex subproblems. Monotonicity of the proposed algorithms are guaranteed and their global convergence to a KKT solution are established. Numerical simulations demonstrate that a substantial performance improvement can be achieved by the proposed algorithms over the benchmarks.

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APPENDIX A
PROOF OF THEOREM 4.2

Suppose the sequence $\{(\mathbf{u}^n, \mathbf{v}^n)\}$ does not terminate finitely, then there must exist an accumulation point for the bounded sequence $\{\|\mathbf{u}^n\|\}$ and $\{\|\mathbf{v}^n\|\}$. Denote the accumulation point as $\bar{\mathbf{u}}$ and $\bar{\mathbf{v}}$, and we have $\mathbf{u}^n \to \bar{\mathbf{u}}$ and $\mathbf{v}^n \to \bar{\mathbf{v}}$ with n from an infinite subsequence $\mathcal{N}$. Without loss of generality, we also assume that $\mathbf{v}^{n+1} \to \hat{\mathbf{v}},\ n \in \mathcal{N}$. Furthermore, the monotonicity of $\{G_n\}$ (according to the remark after problem (22)) implies $G_n \to G(\bar{\mathbf{u}}, \hat{\mathbf{v}}) = G(\bar{\mathbf{u}}, \bar{\mathbf{v}}) \triangleq \bar{G}$ and $G_n \leq \bar{G}$ for all $n \geq 0$. Taking limits from both sides of (23) yields

$$\theta^{n+1} \to \hat{\theta} \triangleq \min_{k \in \mathcal{K}} \left\{ D_k(\bar{\mathbf{u}}, \hat{\mathbf{v}}) - \sqrt{\bar{G}}\, I_k(\bar{\mathbf{u}}, \hat{\mathbf{v}}) \right\} \geq 0,\ n \in \mathcal{N}. \tag{25}$$

We know from the proof of Theorem 4.1 that to show $(\bar{\mathbf{u}}, \bar{\mathbf{v}})$ is a stationary point of problem (2), it is sufficient to prove $\bar{\mathbf{u}} \in \Phi(\bar{\mathbf{v}})$ and $\bar{\mathbf{v}} \in \Psi(\bar{\mathbf{u}})$. The same argument as in the proof of Theorem 4.1 shows that $\bar{\mathbf{u}} \in \Phi(\bar{\mathbf{v}})$. Since $G(\bar{\mathbf{u}}, \hat{\mathbf{v}}) = G(\bar{\mathbf{u}}, \bar{\mathbf{v}})$, it follows that $\bar{\mathbf{v}} \in \Psi(\bar{\mathbf{u}})$ if and only if $\hat{\mathbf{v}} \in \Psi(\bar{\mathbf{u}})$. Next, we establish the claim $\hat{\mathbf{v}} \in \Psi(\bar{\mathbf{u}})$ by proving the following two statements:

1) $(\hat{\mathbf{v}}, \hat{\theta})$ solves the SOCP problem

$$\begin{aligned} \max_{\{\mathbf{v},\theta\}} \quad & \theta \\ \text{s.t.} \quad & \frac{(\bar{\mathbf{u}}_k)^\dagger \mathbf{H}_{kk}\mathbf{v}_k - \theta}{\sqrt{\sigma_k^2 + \sum_{j\neq k} |(\bar{\mathbf{u}}_k)^\dagger \mathbf{H}_{kj}\mathbf{v}_j|^2}} \geq \sqrt{\bar{G}},\, k \in \mathcal{K}, \\ & \|\mathbf{v}_k\|^2 \leq P_k,\, k \in \mathcal{K}; \end{aligned}$$

2) $\hat{\theta} = 0$.

In fact, for any feasible $\mathbf{v}$, due to the optimality of $\mathbf{v}^{n+1}$, it follows for all $n \in \mathcal{N}$,

$$\min_{k\in\mathcal{K}} \left\{ D_k(\mathbf{u}^n, \mathbf{v}^{n+1}) - \sqrt{G_{2n}}\, I_k(\mathbf{u}^n, \mathbf{v}^{n+1}) \right\} \geq \min_{k\in\mathcal{K}} \left\{ D_k(\mathbf{u}^n, \mathbf{v}) - \sqrt{G_{2n}}\, I_k(\mathbf{u}^n, \mathbf{v}) \right\}. \tag{26}$$

Taking limits from both sides of (26) and recalling the definition of $\hat{\theta}$ in (25), we obtain

$$\hat{\theta} \geq \min_{k\in\mathcal{K}} \left\{ D_k(\bar{\mathbf{u}}, \mathbf{v}) - \sqrt{\bar{G}}\, I_k(\bar{\mathbf{u}}, \mathbf{v}) \right\}. \tag{27}$$

Hence the first statement holds true. Now we show the second statement $\hat{\theta} = 0$. Since $G_{2n+1} \leq \bar{G}$ and

$$\min_{k\in\mathcal{K}} \left\{ D_k(\mathbf{u}^n, \mathbf{v}^{n+1}) - \sqrt{G_{2n+1}}\, I_k(\mathbf{u}^n, \mathbf{v}^{n+1}) \right\} = 0,$$

we have

$$\min_{k\in\mathcal{K}} \left\{ D_k(\mathbf{u}^n, \mathbf{v}^{n+1}) - \sqrt{\bar{G}}\, I_k(\mathbf{u}^n, \mathbf{v}^{n+1}) \right\} \leq 0,\ n \in \mathcal{N}.$$

Taking limits in the above, we know that $\hat{\theta} \leq 0$. Combining this and (25) yields $\hat{\theta} = 0$. This completes the proof.

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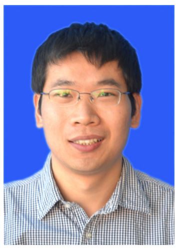

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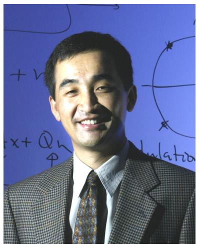

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