Joint Power and Admission Control via Linear Programming Deflation

Ya-Feng Liu, Yu-Hong Dai, and Zhi-Quan Luo

Abstract

We consider the joint power and admission control problem for a wireless network consisting of multiple interfering links. The goal is to support a maximum number of links at their specified signal to interference plus noise ratio (SINR) targets while using a minimum total transmission power. In this work, we first reformulate this NP-hard problem as a sparse ℓ_0-minimization problem and then relax it to a linear program. Furthermore, we derive two easy-to-check necessary conditions for all links in the network to be simultaneously supported at their target SINR levels, and use them to iteratively remove strong interfering links (deflation). An upper bound on the maximum number of supported links is also given. Numerical simulations show that the proposed approach compares favorably with the existing approaches in terms of the number of supported links, the total transmission power, and the execution time.

Index Terms

Admission control, convex approximation, link removal, power control, sparse optimization.

 This work is supported in part by the China National Funds for Distinguished Young Scientists, Grant 11125107, the National Natural Science Foundation, Grant 10831006, the CAS Grant kjcx-yw-s7-03, and in part by the National Science Foundation, Grant DMS-1015346. Part of this work was presented in the IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP), Kyoto, Japan, March 25-30, 2012 [20]. This work was completed during a visit of the first author to the University of Minnesota.

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I. Introduction

Power and admission control are effective tools for interference management in cellular, ad hoc, and cognitive underlay wireless networks [1]–[8]. Conventionally, the processes of admission and power control are decoupled. In particular, users are first admitted to the network based on the available resources. Once some users are admitted, a power control procedure is invoked to reset the transmission power levels so that all interfering links can be supported at their desired signal to interference plus noise ratio (SINR) levels with minimum total transmission power. However, when the network experiences strong interference, not all new and existing links can be simultaneously supported regardless of power control used. In this case, it is necessary to remove some links since their desired service levels can no longer be met due to mutual interference. Unfortunately, the decoupling of the admission and power control steps often leads to unnecessary removal of many links. A more beneficial approach is to combine the admission and power control steps so as to simultaneously determine the maximum number of concurrently supportable links and the corresponding optimal power allocation for each transmitter. Furthermore, joint power and admission control can determine which interfering links must be turned off and rescheduled along orthogonal resource dimensions (such as time, space or frequency slots).

Joint power and admission control can alleviate the difficulties associated with the convergence of stand-alone power control methods. For example, consider the well-known distributed power control strategy by Foschini-Miljanic algorithm [4] where at each step, every transmitter independently updates its power level by a multiplicative factor equal to the ratio of its target SINR level and its measured SINR level. In this way, each transmitter increases its power level if its measured SINR value does not reach its SINR target (note that in this case the multiplicative factor is strictly greater than one) and otherwise decreases its power level. For any prespecified SINR levels that are feasible, this power control strategy has been shown [4] to converge geometrically to a solution that supports all the links at the given SINR targets with minimum total transmission power. A longstanding issue associated with this power control strategy is that it does not converge when the preselected SINR levels are *infeasible*, i.e., not all links in the network can be simultaneously supported at their SINR targets. In this case, we must adopt a joint power and admission control approach to determine which links should be removed. In this paper, we propose an efficient way to *selectively remove links* so that the remaining ones can be simultaneously supported at their desired SINR levels. The goal is to maximize the number of links simultaneously supportable at their required SINR targets while using minimum total transmission power.

A. Related Work

The joint power and admission control problem can be solved to global optimality by checking the simultaneous supportability of every subset of links. However, the computational complexity of this enumeration approach grows exponentially with the total number of links. Another global optimal algorithm based on the branch and bound strategy is considered in [9]. Theoretically, the problem is known to be NP-hard [1], [3], so various heuristic algorithms have been proposed for this problem.

In [10] and [11], the author proposed a centralized stepwise removal algorithm (SRA) and a distributed limited information stepwise removal algorithm (LI-SRA) respectively. Some extensions of [10] and [11] were reported in [12]–[14]. The joint power and admission control algorithms proposed in these references do not assume any power constraints. Assuming individual power constraints, the reference [3] proposed a gradual removal non-restricted distributed constrained power control (GRN-DCPC) algorithm in which the power is updated by a modified version of Foschini-Miljanic algorithm, i.e., each link's power is updated by taking a minimum between the one given by the Foschini-Miljanic algorithm and its power budget. Whenever a certain link's power level given by the Foschini-Miljanic algorithm exceeds its power budget, we remove the link that has the largest interference plus noise footprint (called SMART removal rule in [3]). The removal procedure is terminated until all the remaining links in the network can be simultaneously supported.

Convex approximation algorithms for joint beamforming and admission control have been proposed in [15] for a cellular downlink network. The techniques were further extended in [1] to the joint power and admission control problem in cognitive underlay networks. Instead of directly solving the original NP-hard problem, the idea of the proposed linear programming deflation (LPD) algorithm [1] is to approximate the problem by an appropriate LP, whose solution can be used to iteratively remove interfering links (the LP approximation and the removal strategy used in [1] will be given late). Again, the removal procedure is terminated if all the remaining links in the network are simultaneously supportable.

A recent work [2] proposed another removal-based heuristic algorithm for the joint power and admission control problem. Assume that each link has the same SINR target. At each step, the link that results in the largest increase in the achievable SINR is removed until all links in the network are simultaneously supportable. The above idea is *approximately* implemented in the Algorithm II-B [2] to reduce the computational complexity. See [9], [16]–[21] for other results on joint power and admission control.

B. Our Contribution

In this paper, we show that the joint power and admission control problem can be equivalently reformulated as a sparse ℓ_0-minimization problem. We then use the ℓ_1-relaxation to derive a linear program (different from that in [1]) whose solution can be used to check the simultaneous supportability of all links in the network and to guide an iterative link removal procedure (deflation). We also develop two easily checkable necessary conditions for all links in the network to be simultaneously supported at their desired SINR requirements. These conditions allow us to iteratively remove strong interfering links and therefore significantly accelerate the deflation process. Numerical results show that the proposed algorithm outperforms the existing approaches in [1], [2] in terms of both the number of supported links and the CPU time.

C. Notations

We adopt the following notations in this paper. We denote the index set $\{1,2,\cdots,K\}$ by $\mathcal{K}$. Lowercase boldface and uppercase boldface are used for vectors and matrices, respectively. For a given vector $\mathbf{x}$, the notations $\max\{\mathbf{x}\}$, $(\mathbf{x})_k$, and $\|\mathbf{x}\|_0$ stand for its maximum entry, its k-th entry, and the number of its nonzero entries, respectively. We use $\mathbf{x}_1 \circ \mathbf{x}_2$ to represent the Hadamard product of two vectors $\mathbf{x}_1$ and $\mathbf{x}_2$. For any subset $\mathcal{I} \subseteq \mathcal{K}$, we use $\mathbf{A}_{\mathcal{I}}$ to denote the matrix formed by the rows of $\mathbf{A}$ indexed by $\mathcal{I}$. For a vector $\mathbf{x}$, the notation $\mathbf{x}_{\mathcal{I}}$ is similarly defined. Moreover, for any $\mathcal{J} \subseteq \mathcal{K}$, the notation $\mathbf{A}_{\mathcal{I}\mathcal{J}}$ will denote the submatrix of $\mathbf{A}$ obtained by taking the rows and columns of $\mathbf{A}$ indexed by $\mathcal{I}$ and $\mathcal{J}$ respectively. The spectral radius of a matrix $\mathbf{A}$ is denoted by $\rho(\mathbf{A})$. Finally, we use $\mathbf{e}$ to represent the vector with all components being one and $\mathbf{I}$ to represent the identity matrix of an appropriate size, respectively.

II. Problem Formulation

Consider a K-link single-input single-output (SISO) interference channel with channel gains $g_{kj} \geq 0$ (from the transmitter of link j to the receiver of link k), noise power $\eta_k > 0$, SINR target $\gamma_k > 0$, and power budget $p_k^{\max} > 0$ for $k, j \in \mathcal{K} := \{1,2,\cdots,K\}$. Denote the power allocation vector by $\mathbf{p} = (p_1, p_2, \cdots, p_K)^T$ and the power budget vector by $\mathbf{p}^{\max} = (p_1^{\max}, p_2^{\max}, \cdots, p_K^{\max})^T$. The joint power and admission control problem can be mathematically formulated as a two-stage optimization problem. Specifically, the first stage maximizes the number of admitted links:

$$
\begin{aligned}
\max_{\mathbf{p},\,\mathcal{S}} \quad & |\mathcal{S}| \\
\text{s.t.} \quad & \text{SINR}_k \geq \gamma_k,\ k \in \mathcal{S} \subseteq \mathcal{K}, \\
& \mathbf{0} \leq \mathbf{p} \leq \mathbf{p}^{\max},
\end{aligned}
\tag{1}
$$

where $|\mathcal{S}|$ denotes the cardinality of the set $\mathcal{S}$, and the SINR value at the k-th receiver is

$$\text{SINR}_k = \frac{g_{kk} p_k}{\eta_k + \sum_{j \neq k} g_{kj} p_j}.$$

We use $\mathcal{S}_0$ to denote the *maximum admissible set* for problem (1). Notice that the maximum admissible set $\mathcal{S}_0$ might not be unique. The second stage minimizes the total transmission power required to support these admitted links in $\mathcal{S}_0$:

$$\begin{aligned} \min_{\{p_k\}_{k\in\mathcal{S}_0}} \quad & \textstyle\sum_{k\in\mathcal{S}_0} p_k \\ \text{s.t.} \quad & \text{SINR}_k \geq \gamma_k,\ k \in \mathcal{S}_0, \\ & 0 \leq p_k \leq p_k^{\max},\ k \in \mathcal{S}_0. \end{aligned} \tag{2}$$

Due to the choice of $\mathcal{S}_0$, power control problem (2) is feasible and can be efficiently solved by the Foschini-Miljanic algorithm [4].

III. A New Linear Programming Deflation Algorithm

Consider the K-link SISO interference channel model introduced in Section II. To facilitate the development of the new linear programming deflation (NLPD) algorithm, we first normalize the channel parameters to obtain an equivalent normalized channel. In particular, let us use $\mathbf{q} = (q_1, q_2, \cdots, q_K)^T$ with

$$q_k = \frac{p_k}{p_k^{\max}}, \quad \forall\ k \in \mathcal{K} \tag{3}$$

to denote the normalized power allocation vector, and use $\mathbf{c} = (c_1, c_2, \cdots, c_K)^T$ with

$$c_k = \frac{\gamma_k \eta_k}{g_{kk} p_k^{\max}} > 0, \quad \forall\ k \in \mathcal{K}$$

to denote the normalized noise vector. It is obvious that $\mathbf{p} = \mathbf{p}^{\max} \circ \mathbf{q}$. We define a normalized channel matrix $\mathbf{A} \in \mathbb{R}^{K\times K}$ by

$$a_{kj} = \begin{cases} 1, & \text{if } k = j, \\ -\dfrac{\gamma_k g_{kj} p_j^{\max}}{g_{kk} p_k^{\max}}, & \text{if } k \neq j, \end{cases} \tag{4}$$

where a_{kj} denotes the (k, j)-th entry of $\mathbf{A}$, and $|a_{kj}|$ is the normalized channel gain from the transmitter of link j to the receiver of link k. Notice that the matrix $\mathbf{A}$ is a square matrix with diagonal entries equal to one and nonpositive off-diagonal entries. This special structure of $\mathbf{A}$ will play an important role in the development of the NLPD algorithm.

With this normalization, we can see that

$$
\begin{aligned}
(\mathbf{A}\mathbf{q}-\mathbf{c})_k &= q_k + \sum_{j\neq k} a_{kj} q_j - c_k \\
&= \frac{p_k}{p_k^{\max}} - \sum_{j\neq k}\left(\frac{\gamma_k g_{kj} p_j^{\max}}{g_{kk} p_k^{\max}}\right)\left(\frac{p_j}{p_j^{\max}}\right) - \frac{\gamma_k \eta_k}{g_{kk} p_k^{\max}} \\
&= \frac{1}{g_{kk} p_k^{\max}}\left(\eta_k + \sum_{j\neq k} g_{kj} p_j\right)\left(\frac{g_{kk} p_k}{\eta_k + \sum_{j\neq k} g_{kj} p_j} - \gamma_k\right).
\end{aligned}
$$

Thus, link k is supported at its desired SINR level

$$
\text{SINR}_k = \frac{g_{kk} p_k}{\eta_k + \sum_{j\neq k} g_{kj} p_j} \geq \gamma_k
$$

if and only if

$$
(\mathbf{A}\mathbf{q}-\mathbf{c})_k \geq 0.
$$

Consequently, problem (1) can be equivalently rewritten as

$$
\begin{aligned}
\max_{\mathbf{q},\mathcal{S}} \quad & |\mathcal{S}| \\
\text{s.t.} \quad & (\mathbf{A}\mathbf{q}-\mathbf{c})_k \geq 0,\ k \in \mathcal{S} \subseteq \mathcal{K}, \\
& \mathbf{0} \leq \mathbf{q} \leq \mathbf{e},
\end{aligned} \tag{5}
$$

and problem (2) can be restated as

$$
\begin{aligned}
\min_{\{q_k\}_{k\in\mathcal{S}_0}} \quad & \sum_{k\in\mathcal{S}_0} p_k^{\max} q_k \\
\text{s.t.} \quad & (\mathbf{A}\mathbf{q}-\mathbf{c})_k \geq 0,\ k \in \mathcal{S}_0, \\
& 0 \leq q_k \leq 1,\ k \in \mathcal{S}_0.
\end{aligned} \tag{6}
$$

A. ℓ_0-Minimization Reformulation

We now formulate the two-stage admission/power control problem (5) and (6) as a single-stage ℓ_0-minimization problem. To derive this reformulation, we need to use the following balancing lemma, which was first proposed in [22] and later studied further in [10], [23]. It is a consequence of the positivity of vector $\mathbf{c}$ and the special form of matrix $\mathbf{A}$.

Lemma 1 (Balancing Lemma): Let $\mathbf{A}$ be a square matrix whose diagonal entries are equal to one and off-diagonal entries are nonpositive. Let $\mathbf{c}$ be a vector with positive entries. Suppose that $\mathbf{A}\tilde{\mathbf{q}} \geq \mathbf{c}$ for some $\tilde{\mathbf{q}} \geq \mathbf{0}$. Then there exists a vector $\bar{\mathbf{q}}$ satisfying $\mathbf{c} \leq \bar{\mathbf{q}} \leq \tilde{\mathbf{q}}$ and $\mathbf{A}\bar{\mathbf{q}} = \mathbf{c}$. In addition, for any $\boldsymbol{\theta} > \mathbf{0}$, the vector $\bar{\mathbf{q}}$ solves the optimization problem

$$\begin{aligned} \min_{\mathbf{q}} \quad & \boldsymbol{\theta}^T \mathbf{q} \\ \text{s.t.} \quad & \mathbf{A}\mathbf{q} \geq \mathbf{c}, \\ & \mathbf{q} \geq \mathbf{0}. \end{aligned}$$

Lemma 1 implies that if all links can be supported with a power allocation $\mathbf{0} \leq \mathbf{q} \leq \mathbf{e}$, then the minimum total power allocation must support all links in the network exactly at their target SINR levels. Supporting any link above its target SINR level will mean a waste of the total transmission power.

Using Lemma 1, we can reformulate the two-stage joint power and admission control problem (1) and (2) (equivalent to problem (5) and (6)) as a single-stage sparse optimization problem

$$\begin{aligned} \min_{\mathbf{x},\,\mathbf{q}} \quad & \|\mathbf{x}\|_0 + \alpha \left(\mathbf{p}^{\max}\right)^T \mathbf{q} \\ \text{s.t.} \quad & \mathbf{x} = \mathbf{A}\mathbf{q} - \mathbf{c}, \\ & \mathbf{0} \leq \mathbf{q} \leq \mathbf{e}, \end{aligned} \tag{7}$$

where α is a constant satisfying

$$0 < \alpha < \alpha_1 := \frac{1}{\left(\mathbf{p}^{\max}\right)^T \mathbf{e}}. \tag{8}$$

Actually, if there are more than one maximum admissible set (i.e., the solution for problem (1) is not unique), the formulation (7) is capable of picking the one with *minimum* total transmission power as a result of the second term in the objective of (7).

Theorem 1: Suppose $(\mathbf{x}^*, \mathbf{q}^*)$ is the solution to problem (7). Then the optimal value of problem (1) is M if and only if $\|\mathbf{x}^*\|_0 = K - M$. In fact, the set of links indexed by $\{\ k \in \mathcal{K} \mid x_k^* = 0\ \}$ (whose cardinality is M) is simultaneously supportable by the power allocation $\mathbf{p}^* = \mathbf{p}^{\max} \circ \mathbf{q}^*$ in the original channel. Moreover, $\left(\mathbf{p}^{\max}\right)^T \mathbf{q}^*$ is the minimum total transmission power required to support any M links in the network.

We prove Theorem 1 by first establishing the equivalence of (1) with the following intermediate problem

$$\begin{aligned} \min_{\mathbf{x},\,\mathbf{q}} \quad & \|\mathbf{x}\|_0 \\ \text{s.t.} \quad & \mathbf{x} = \mathbf{A}\mathbf{q} - \mathbf{c}, \\ & \mathbf{0} \leq \mathbf{q} \leq \mathbf{e}, \end{aligned} \tag{9}$$

and then proving the equivalence of this intermediate problem with (7). We relegate the proof of Theorem 1 to Appendix A.

It is shown in [1] that the joint power and admission control problem (1) and (2) can be equivalently formulated as the following integer program

$$\begin{aligned}\min_{\mathbf{p},\,\mathbf{t}} \quad & \epsilon\sum_{k\in\mathcal{K}} p_k + (1-\epsilon)\sum_{k\in\mathcal{K}} t_k \\ \text{s.t.} \quad & \frac{g_{kk}p_k + \delta_k^{-1}t_k}{\eta_k + \sum_{j\neq k} g_{kj}p_j} \geq \gamma_k,\ k\in\mathcal{K}, \\ & t_k \in \{0,\,4\}\,,\ k\in\mathcal{K}, \\ & \mathbf{0}\leq \mathbf{p} \leq \mathbf{p}^{\max},\end{aligned} \tag{10}$$

where the parameters ϵ and δ_k $(k\in\mathcal{K})$ satisfy

$$0<\epsilon<\frac{4}{\mathbf{e}^T\mathbf{p}^{\max}+4}, \tag{11}$$

$$\delta_k \leq \frac{4}{\gamma_k\left(\sum_{j\neq k} g_{kj}p_j^{\max}+\eta_k\right)}. \tag{12}$$

In (10), $t_k\in\{0,\,4\}$ is the admission variable of link k ($t_k=0$ means link k is admitted while $t_k=4$ means link k is dropped), ϵ is used to prioritize the admission control term $(\sum_{k\in\mathcal{K}} t_k)$ over the power control term $(\sum_{k\in\mathcal{K}} p_k)$, and δ_k $(k\in\mathcal{K})$ are some small positive constants used to guarantee the feasibility of problem (10).

Interestingly, the above formulation (10) can be viewed as a sparse optimization problem. Notice that the solution $(\mathbf{p}^*,\mathbf{t}^*)$ to problem (10) satisfies

$$\frac{g_{kk}p_k^* + \delta_k^{-1}t_k^*}{\eta_k + \sum_{j\neq k} g_{kj}p_j^*} = \gamma_k,\ k\in\mathcal{K},$$

so we have

$$t_k^* = \delta_k\left(\gamma_k(\eta_k + \sum_{j\neq k} g_{kj}p_j^*) - g_{kk}p_k^*\right) \in \{0,\,4\}\,,\ k\in\mathcal{K},$$

which further implies that

$$\|\mathbf{t}\|_0 = \|\mathbf{c}-\mathbf{A}\mathbf{q}\|_0 = \|\mathbf{x}\|_0.$$

Therefore, the vector $\mathbf{t}$ in (10) plays the similar role as the vector $\mathbf{x}$ in (7).

The ℓ_0-norm reformulation (7) has a discontinuous objective function due to the first term $\|\mathbf{x}\|_0$. Actually, the ℓ_0-minimization problem (7) is NP-hard. This follows from Theorem 1, which says problem (7) is equivalent to the joint problem (1) and (2), and the fact that problem (1) is NP-hard [1]. The NP-hardness proof in [1] is based on a polynomial-time reduction from the maximum independent set problem. Moreover, we know from [24] that for a K-node graph, there does not exist a polynomial-time K^{-c}-approximate algorithm for the maximum independent set problem with constant $c>0$. Therefore, problem (7) is not only hard to solve (to global optimality) but also hard to approximate (to constant factor global optimality). However, the reformulation (7) of the joint power and admission control problem allows for a simple convex relaxation.

B. Linear Programming Relaxation

Since ℓ_0-optimization problem (7) is still NP-hard, it is natural to consider its ℓ_1-convex relaxation

$$\begin{aligned} \min_{\mathbf{x},\,\mathbf{q}} \quad & \|\mathbf{x}\|_1 + \alpha\,(\mathbf{p}^{\max})^T\,\mathbf{q} \\ \text{s.t.} \quad & \mathbf{x} = \mathbf{A}\mathbf{q} - \mathbf{c}, \\ & \mathbf{0} \le \mathbf{q} \le \mathbf{e}. \end{aligned} \tag{13}$$

By introducing auxiliary variables and additional constraints, the above problem (13) is easily converted to a linear program (LP). Interestingly, by exploiting the special structure of $\mathbf{A}$, we can convert (13) to an equivalent LP (14) without using any auxiliary variables. The proof of this result (Theorem 2) can be found in Appendix B.

Theorem 2: The ℓ_1-relaxation problem (13) is *equivalent* to the following linear program

$$\begin{aligned} \min_{\mathbf{q}} \quad & \mathbf{e}^T\,(\mathbf{c} - \mathbf{A}\mathbf{q}) + \alpha\,(\mathbf{p}^{\max})^T\,\mathbf{q} \\ \text{s.t.} \quad & \mathbf{c} - \mathbf{A}\mathbf{q} \ge \mathbf{0}, \\ & \mathbf{0} \le \mathbf{q} \le \mathbf{e}. \end{aligned} \tag{14}$$

We now compare the LP approximation (14) with the one used in [1]. The LP approximation in [1]

$$\begin{aligned} \min_{\mathbf{p},\,\mathbf{t}} \quad & \epsilon \sum_{k\in\mathcal{K}} p_k + (1-\epsilon) \sum_{k\in\mathcal{K}} t_k \\ \text{s.t.} \quad & \frac{g_{kk}p_k + \delta_k^{-1} t_k}{\eta_k + \sum_{j\neq k} g_{kj}p_j} \ge \gamma_k,\ k \in \mathcal{K}, \\ & 0 \le t_k \le 4,\ k \in \mathcal{K}, \\ & \mathbf{0} \le \mathbf{p} \le \mathbf{p}^{\max}, \end{aligned} \tag{15}$$

is obtained by relaxing the integer constraint $t_k \in \{0,\,4\}$ in (10) to the linear constraint $t_k \in [0,\,4]$. From the forms of LP approximations (14) and (15), we can see that the *reversed* SINR constraints make problem (14) always feasible ($\mathbf{q} = \mathbf{0}$ is a certificate for the feasibility), whereas extra parameters δ_k $(k \in \mathcal{K})$ need be chosen to guarantee the feasibility of problem (15). In addition, the introduction of auxiliary admission variables t_k $(k \in \mathcal{K})$ makes the number of unknown variables in problem (15) twice as large as that in (14). Further, for any $\mathbf{e} \ge \mathbf{q} \ge \mathbf{0}$ satisfying $\mathbf{c} - \mathbf{A}\mathbf{q} \ge 0$, we define

$$q_k^e = (\mathbf{c} - \mathbf{A}\mathbf{q})_k\,, \quad \forall\ k \in \mathcal{K}. \tag{16}$$

We know from [1] that q_k^e measures the excess transmission power that the transmitter of link k needs in the normalized channel in order to be served with its desired SINR target, assuming all other links keep their transmission powers unchanged. Therefore, LP (14) actually minimizes a weighted sum of the total excess transmission power $\mathbf{e}^T(\mathbf{c}-\mathbf{A}\mathbf{q})$ and the total real transmission power $(\mathbf{p}^{\max})^T\mathbf{q}$.

In general, we can consider the weighted ℓ_1-convex approximation of problem (7) as follows:

$$\begin{aligned} \min_{\mathbf{x},\,\mathbf{q}} \quad & \|\mathbf{w}\circ\mathbf{x}\|_1 + \alpha\,(\mathbf{p}^{\max})^T\mathbf{q} \\ \text{s.t.} \quad & \mathbf{x} = \mathbf{A}\mathbf{q}-\mathbf{c}, \\ & \mathbf{0}\le\mathbf{q}\le\mathbf{e}, \end{aligned} \tag{17}$$

where $\mathbf{w}$ is a nonnegative weight vector. The ideal weight vector $\mathbf{w}$ should be

$$w_k = \begin{cases} 1, & \text{if } k\in\mathcal{K}^*, \\ 0, & \text{if } k\notin\mathcal{K}^*, \end{cases}$$

where $\mathcal{K}^*$ is the optimal maximum admissible set of problem (7). If we choose the weight vector $\mathbf{w}$ in this way, then the solution to problem (17) solves the ℓ_0-minimization problem (7). In a similar manner to problem (13), problem (17) can be transformed into an LP without introducing any auxiliary variables.

It is worth pointing out that the LP approximation (15) used in [1] can be seen as a weighted ℓ_1-relaxation of problem (7). Since the solution $(\mathbf{p}^*,\mathbf{t}^*)$ to problem (15) satisfies

$$\frac{g_{kk}p_k^* + \delta_k^{-1}t_k^*}{\eta_k+\sum_{j\neq k}g_{kj}p_j^*} = \gamma_k,\ k\in\mathcal{K},$$

we have

$$t_k^* = \delta_k\left(\gamma_k(\eta_k+\sum_{j\neq k}g_{kj}p_j^*) - g_{kk}p_k^*\right)\ge 0,\ k\in\mathcal{K}.$$

Recall the choice of δ_k in (12), it follows that $t_k^*\le 4,\ k\in\mathcal{K}$. Therefore, problem (15) is equivalent to

$$\begin{aligned} \min_{\mathbf{p}} \quad & \epsilon\sum_{k\in\mathcal{K}}p_k + (1-\epsilon)\sum_{k\in\mathcal{K}}\delta_k\left(\gamma_k(\eta_k+\sum_{j\neq k}g_{kj}p_j) - g_{kk}p_k\right) \\ \text{s.t.} \quad & \gamma_k(\eta_k+\sum_{j\neq k}g_{kj}p_j) - g_{kk}p_k \ge 0,\ k\in\mathcal{K}, \\ & \mathbf{0}\le\mathbf{p}\le\mathbf{p}^{\max}, \end{aligned} \tag{18}$$

which is further equivalent to problem (17) with

$$w_k = \delta_k g_{kk} p_k^{\max}\ (k\in\mathcal{K}) \text{ and } \alpha = \frac{\epsilon}{1-\epsilon}.$$

If we rewrite the LP relaxation (14) in terms of the original channel parameters, we obtain

$$\begin{aligned} \min_{\mathbf{p}} \quad & \alpha \sum_{k\in\mathcal{K}} p_k + \sum_{k\in\mathcal{K}} \frac{1}{g_{kk}p_k^{\max}} \left(\gamma_k(\eta_k + \sum_{j\neq k} g_{kj}p_j) - g_{kk}p_k\right) \\ \text{s.t.} \quad & \gamma_k(\eta_k + \sum_{j\neq k} g_{kj}p_j) - g_{kk}p_k \geq 0, \;\; k\in\mathcal{K}, \\ & \mathbf{0} \leq \mathbf{p} \leq \mathbf{p}^{\max}, \end{aligned} \tag{19}$$

which is equivalent to problem (17) with $\mathbf{w} = \mathbf{e}$. Although both of (18) and (19) can be viewed as weighted linear programming approximations of problem (7), they are technically different. The chosen weight vector $\mathbf{w}$ plays the key role in the approximation performance. In (19), we set $w_k = 1$ (treating all links equally), which is natural as all links's direct-channel gains, power budgets, and SINR targets are one in the normalized channel. We also notice that

$$1 \leq \frac{4g_{kk}p_k^{\max}}{\gamma_k\left(\sum_{j\neq k} g_{kj}p_j^{\max} + \eta_k\right)} \tag{20}$$

may not hold true (the left hand side of (20) is our chosen weight, while the right hand side of (20) is the upper bound of the chosen weight in [1]). This means our choice of the weight vector is not a special one of (18).

We now discuss the choice of parameter α in (14). If there exists some vector $\bar{\mathbf{q}}$ such that $\mathbf{0} \leq \bar{\mathbf{q}} \leq \mathbf{e}$ and $\mathbf{A}\bar{\mathbf{q}} = \mathbf{c}$, then $\rho(\mathbf{I} - \mathbf{A}) < 1$ [22]. Thus $\mathbf{A}$ is nonsingular and $\mathbf{A}^{-1}$ is nonnegative [25, Theorem 1.15], which further implies that

$$\mathbf{z} = (\mathbf{A}^T)^{-1}\mathbf{p}^{\max} > \mathbf{0}. \tag{21}$$

Let us define

$$\alpha_2 := \frac{1}{\max\{\mathbf{z}\}} > 0. \tag{22}$$

By checking the KKT condition of (14), we can see that the vector $\bar{\mathbf{q}}$ (satisfying $\mathbf{0} \leq \bar{\mathbf{q}} \leq \mathbf{e}$ and $\mathbf{A}\bar{\mathbf{q}} = \mathbf{c}$) solves LP (14) provided that $0 \leq \alpha \leq \alpha_2$. This shows that the solution to LP (14) with $0 \leq \alpha \leq \alpha_2$ can simultaneously support all links at their desired SINR targets (will not over remove links) as long as all links in the network are simultaneously supportable. Combining (8) and (22), we propose to choose the parameter α in (14) according to

$$\alpha = \begin{cases} c_1\alpha_1, & \text{if } \rho(\mathbf{I} - \mathbf{A}) \geq 1, \\ c_2 \min\{\alpha_1, \alpha_2\}, & \text{if } \rho(\mathbf{I} - \mathbf{A}) < 1, \end{cases} \tag{23}$$

where $0 < c_1 \leq c_2 < 1$ are two constants. The motivation of the choice (23) is that, if $\rho(\mathbf{I} - \mathbf{A}) \geq 1$, in which case it is not possible to simultaneously support all links in the network, a smaller α is preferable to give more priority to the admission control term so that more links could be supported.

The solution of LP (14) can be used to guide our link removal process. In particular, by solving (14) with α given in (23), we know whether all links in the network can be simultaneously supported by simply checking if the solution $\mathbf{q}$ satisfies $\mathbf{A}\mathbf{q} = \mathbf{c}$. Furthermore, having obtained the solution of (14), we can use the efficient removal strategy in [1] to drop the link k_0 defined by

$$k_0 = \arg\max_{k\in\mathcal{K}} \left\{ \sum_{j\neq k} |a_{jk}| q_k^e + \sum_{j\neq k} |a_{kj}| q_j^e \right\}, \tag{24}$$

where a_{kj} and q_k^e are defined in (4) and (16), respectively.

Let us rewrite the removal strategy (24) in terms of the original channel parameters. Define the excess transmission power p_k^e in the original channel [1] as

$$p_k^e = \frac{\gamma_k \left(\eta_k + \sum_{j\neq k} g_{kj} p_j\right) - g_{kk} p_k}{g_{kk}}.$$

By (3) and (16), we can relate p_k^e to q_k^e as follows,

$$p_k^e = p_k^{\max}\left(p_k^e / p_k^{\max}\right) = p_k^{\max}(\mathbf{c} - \mathbf{A}\mathbf{q})_k = p_k^{\max} q_k^e.$$

Consequently, the removal strategy (24) can be rewritten as

$$k_0 = \arg\max_{k\in\mathcal{K}} \left\{ \sum_{j\neq k} \frac{\gamma_j}{g_{jj} p_j^{\max}} g_{jk} p_k^e + \sum_{j\neq k} \frac{\gamma_k}{g_{kk} p_k^{\max}} g_{kj} p_j^e \right\}. \tag{25}$$

Therefore, due to the presence of normalization, the proposed removal strategy (24) (equivalent to (25)) is different from the one used in [1] which is given by

$$k_0 = \arg\max_{k\in\mathcal{K}} \left\{ \sum_{j\neq k} g_{jk} p_k^e + \sum_{j\neq k} g_{kj} p_j^e \right\}. \tag{26}$$

Comparing (25) and (26), we can see that the removal strategy (25) takes more factors into consideration, including target SINRs, direct-link channel gains, and power budgets.

Next, we give a sufficient condition for the solution to the LP (14) solves the ℓ_0-minimization problem (7). Without loss of generality, suppose that

$$\mathbf{A} = \begin{pmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{pmatrix}, \ \mathbf{q} = \begin{pmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \end{pmatrix}, \ \mathbf{c} = \begin{pmatrix} \mathbf{c}_1 \\ \mathbf{c}_2 \end{pmatrix},$$

and $\mathbf{q}^* = \begin{pmatrix} \mathbf{q}_1^* \\ \mathbf{0} \end{pmatrix}$ is the solution to the ℓ_0-minimization problem, then we have

$$\mathbf{A}_{11}\mathbf{q}_1^* = \mathbf{c}_1 > \mathbf{0}, \ \mathbf{e} \geq \mathbf{q}_1^* > \mathbf{0}, \ \mathbf{q}_2^* = \mathbf{0}. \tag{27}$$

Proposition 1: The ℓ_0-minimization problem (7) and the LP (14) share the same optimal solution if the following conditions

$$\begin{cases} \mathbf{e} + \left(\mathbf{A}_{11}^T\right)^{-1}\mathbf{A}_{21}^T\mathbf{e} - \alpha\left(\mathbf{A}_{11}^T\right)^{-1}\mathbf{p}_1^{\max} > \mathbf{0} \\ \mathbf{A}_{12}^T\left(\mathbf{A}_{11}^T\right)^{-1}\mathbf{A}_{21}^T\mathbf{e} - \mathbf{A}_{22}^T\mathbf{e} - \alpha\mathbf{A}_{12}^T\left(\mathbf{A}_{11}^T\right)^{-1}\mathbf{p}_1^{\max} + \alpha\mathbf{p}_2^{\max} > \mathbf{0} \end{cases} \tag{28}$$

are satisfied.

The proof of Proposition 1 can be found in Appendix C. To prove Proposition 1, it is sufficient to prove that the solution $\mathbf{q}^*$ to the ℓ_0-minimization problem (7) is the *unique* solution to the LP (14). Notice that if the interference level from group 1 (corresponding to supported links in (27)) to group 2 (corresponding to unsupported links in (27)) is low and the interference level from group 2 to group 1 is high[1] in the sense that the entries in $\mathbf{A}_{21} \leq \mathbf{0}$ are close to zero and the entries in $\mathbf{A}_{12} \leq \mathbf{0}$ are small enough, then the sufficient condition (28) holds automatically and the ℓ_0-minimization and the ℓ_1-minimization are equivalent, as long as we choose α to be sufficiently small. This is intuitively appealing since the links in group 1 cause weak interference to the links in group 2 and the links in group 2 cause strong interference to the links in group 1, we expect that the links in group 1 will be supported simultaneously by transmitting appropriate power and the links in group 2 will be shut down (transmitting zero power).

We remark that in general the solution to the LP approximation (14) does not solve the ℓ_0-minimization problem (7). This is the reason why we do not just use the LP (14) to approximate the sparse optimization problem (7), but instead employ a deflation technique to successively approximate the ℓ_0-optimization problem.

C. A New Linear Programming Deflation Algorithm

The basic idea of the proposed NLPD algorithm is to solve LP (14) and check whether all links can be supported or not; if not, remove a link (mathematically, delete the corresponding row and column of $\mathbf{A}$ and the corresponding entry of $\mathbf{c}$) from the network, and solve a reduced LP (14) again until all the remaining links are supported. To accelerate the deflation procedure (avoid solving too many LPs in the form of (14)), we derive two easy-to-check necessary conditions for all links in the network to be simultaneously supported.

Define

$$\boldsymbol{\mu} = \mathbf{A}^T\mathbf{e},$$

[1] both low and high here are relative to the direct-link channel.

then a necessary condition for all links in the network to be supported is that there exists an index k such that

$$(\boldsymbol{\mu})_k = \left(\mathbf{A}^T\mathbf{e}\right)_k = 1 + \sum_{j \neq k} a_{jk} > 0. \tag{29}$$

This is because otherwise, if $\boldsymbol{\mu} \leq \mathbf{0}$ and the linear system $\mathbf{q} \geq \mathbf{0}$, $\mathbf{A}\mathbf{q} = \mathbf{c}$ is feasible, then it follows from $\mathbf{c} > \mathbf{0}$ that

$$0 \geq \boldsymbol{\mu}^T\mathbf{q} = (\mathbf{A}^T\mathbf{e})^T\mathbf{q} = (\mathbf{A}\mathbf{q})^T\mathbf{e} = \mathbf{c}^T\mathbf{e} > 0,$$

which is a contradiction. Thus, if $\boldsymbol{\mu} \leq \mathbf{0}$, then at least one link in the network cannot be supported. Using condition (29), an upper bound can be derived for the cardinality of the maximum admissible set $\mathcal{S}_0$. Specifically, for any $k \in \mathcal{K}$, denote $\mathcal{R}_k$ to be the *minimum removal set* such that $(\boldsymbol{\mu})_k > 0$. Sort $\{a_{jk}\}_{j\in\mathcal{K}}$ such that

$$a_{k_1k} \leq a_{k_2k} \leq \cdots \leq a_{k_rk} \leq a_{k_{r+1}k} \leq \cdots \leq a_{k_{K-1}k} \leq 0 \leq a_{k_Kk} = 1,$$

and pick r such that $1 + \sum_{j=r+1}^{K-1} a_{k_jk} > 0$ and $1 + \sum_{j=r}^{K-1} a_{k_jk} \leq 0$. Then $\mathcal{R}_k = \{k_1, k_2, \cdots, k_r\}$, and the cardinality of the maximum admissible set $\mathcal{S}_0$ is bounded by

$$|\mathcal{S}_0| \leq K - \min_{k\in\mathcal{K}} \{|\mathcal{R}_k|\}.$$

We can strengthen the necessary condition (29) by involving the noise power $\mathbf{c}$. Suppose that all links can be simultaneously served. Then there exists a vector $\mathbf{q}$ such that $\mathbf{0} \leq \mathbf{q} \leq \mathbf{e}$ and $\mathbf{A}\mathbf{q} = \mathbf{c}$. By the definition of $\mathbf{A}$, we have $\mathbf{q} \geq \mathbf{A}\mathbf{q} = \mathbf{c}$. Denote $\boldsymbol{\mu}_+ = \max\{\boldsymbol{\mu}, \mathbf{0}\}$ and $\boldsymbol{\mu}_- = \max\{-\boldsymbol{\mu}, \mathbf{0}\}$. It is obvious that $\boldsymbol{\mu} = \boldsymbol{\mu}_+ - \boldsymbol{\mu}_-$. Multiplying $\mathbf{e}^T$ from both sides of $\mathbf{A}\mathbf{q} = \mathbf{c}$, we get that $\left(\boldsymbol{\mu}_+ - \boldsymbol{\mu}_-\right)^T\mathbf{q} = \mathbf{e}^T\mathbf{c}$. Moreover, we can obtain

$$\boldsymbol{\mu}_+^T\mathbf{e} \geq \boldsymbol{\mu}_+^T\mathbf{q} = \boldsymbol{\mu}_-^T\mathbf{q} + \mathbf{e}^T\mathbf{c} \geq \left(\boldsymbol{\mu}_- + \mathbf{e}\right)^T\mathbf{c},$$

where the first inequality is due to $\mathbf{q} \leq \mathbf{e}$ and the last one is due to $\mathbf{q} \geq \mathbf{c}$. Therefore, the condition

$$\boldsymbol{\mu}_+^T\mathbf{e} - \left(\boldsymbol{\mu}_- + \mathbf{e}\right)^T\mathbf{c} \geq 0 \tag{30}$$

is necessary for all links in the network to be simultaneously supported. Notice that if the necessary condition (29) is not true, then $\boldsymbol{\mu}_+ = \mathbf{0}$ and (30) will not be satisfied. This implies that the necessary condition (30) is stronger than (29).

We can use the necessary condition (30) in the link removal process. In particular, if (30) is violated, then we should drop at least one link from the network. We propose to remove the link k_0 according to

$$k_0 = \arg\max_{k\in\mathcal{K}} \left\{ \sum_{j\neq k} |a_{kj}| + \sum_{j\neq k} |a_{jk}| + c_k \right\}, \tag{31}$$

which corresponds to applying the SMART rule [3] to the normalized channel and substituting $\mathbf{q} = \mathbf{e}$.

The complete description of the NLPD algorithm, based on the new LP relaxation (14), the new removal strategy (24), and the necessary condition (30), is given as follows.

A New Linear Programming Deflation (NLPD) Algorithm

Step 1. Initialization: Input data $(\mathbf{A}, \mathbf{c}, \mathbf{p}^{\max})$.

Step 2. Preprocessing: Remove link k_0 iteratively according to (31) until condition (30) holds true.

Step 3. Power control: Compute parameter α by (23) and solve LP (14); check whether all links are supported: if yes, go to **Step 5**; else go to **Step 4**.

Step 4. Admission control: Remove link k_0 according to (24), set $\mathcal{K} = \mathcal{K}/\left\{k_0\right\}$, and go to **Step 3**.

Step 5. Postprocessing: Check the removed links for possible admission.

A few remarks on the proposed algorithm are in order. First, *strong interfering links* are successively removed in the preprocessing step. This can accelerate the algorithm (especially for strong interference channels). Second, the computation of α by (23) in the power control step can be simplified. That is, once the nonnegative matrix $\mathbf{I} - \mathbf{A}$ (from the definition of $\mathbf{A}$) satisfies $\rho(\mathbf{I} - \mathbf{A}) < 1$, we do not need to check this spectrum condition any more and can just proceed to solve the linear system (21). This is due to the fact that any principal minor $\bar{\mathbf{X}}$ of $\mathbf{X}$ satisfies $0 \leq \rho(\bar{\mathbf{X}}) \leq \rho(\mathbf{X})$ when $\mathbf{X} \geq \mathbf{0}$ (see [25, Theorem 1.14]). Third, the postprocessing step is an attempt to admit those removed links, which might be able to be supported but are removed in the preprocessing and admission control steps. Specifically, we enumerate all the removed links and admit one of them if it can be supported simultaneously with the already supported links. If there are more than one such candidates, we pick the one such that the minimum total transmission power is needed to simultaneously support it with the already supported links. The postprocessing step is terminated if no such candidate exists. We also remark that the proposed NLPD algorithm can be easily extended to cognitive underlay networks [1] by changing constraints in (14) to

$$\begin{cases} (\mathbf{c} - \mathbf{A}\mathbf{q})_k = 0, \ \forall \ k \in \mathcal{K}_1, \\ (\mathbf{c} - \mathbf{A}\mathbf{q})_k \geq 0, \ \forall \ k \in \mathcal{K}_2, \\ \mathbf{0} \leq \mathbf{q} \leq \mathbf{e}, \end{cases}$$

where $\mathcal{K}_1$ and $\mathcal{K}_2$ denote the set of primary users and secondary users, respectively.

We compare the proposed NLPD algorithm and the LPD algorithm in [1] in terms of the computational complexity needed to *drop one link* from the network. Since both require solving a linear program, their asymptotic complexities are both equal to $O\left(|\mathcal{K}|^{3.5}\right)$ [26], although the LPD algorithm solves a LP with twice as many variables. By comparison, the Algorithm II-B in [2] has a complexity of $O(|\mathcal{K}|^4)$, since it needs to solve $|\mathcal{K}|$ eigenvalue problems [27] to check whether all links in the network can be supported.

Finally, we give an illustrative instance to show the efficiency of the NLPD algorithm based on the newly derived LP approximation (14) and the removal strategy (24) compared to the LPD algorithm [1] based on the LP approximation (15) and the removal strategy (26). The original channel gain matrix $\boldsymbol{G} = [g_{kj}]$, the power budget vector $\mathbf{p}^{\max}$, the SINR target and the noise power at the receiver of link k $(k = 1, 2, 3, 4)$ are

$$\boldsymbol{G} = \begin{pmatrix} 0.05 & 0.008 & 0 & 0.002 \\ 0.02 & 0.4 & 0 & 0.01 \\ 0 & 0 & 0.8 & 0 \\ 0.001 & 0.01 & 0 & 0.05 \end{pmatrix}, \ \mathbf{p}^{\max} = \begin{pmatrix} 55 \\ 7 \\ 3 \\ 55 \end{pmatrix}, \ \gamma_k = 1.6, \ \eta_k = 1.$$

For the above instance, the NLPD algorithm can support 3 links with the total transmission power 41.06, and the corresponding power allocation

$$\mathbf{p} = (0, 5.35, 2.00, 33.71)^T$$

is globally optimal, while the LPD algorithm supports 3 links with the total transmission power 69.21, and the corresponding power allocation vector is

$$\mathbf{p} = (34.12, 0, 2.00, 33.09)^T .$$

We remark that the first link in the above instance is removed by the NLPD algorithm in the admission control step not in the preprocessing step, and therefore the comparison of the NLPD algorithm and the LPD algorithm is fair.

IV. Numerical Simulations

We now present some numerical simulation results to illustrate the effectiveness of the proposed NLPD algorithm. In our simulations, we generate the channel parameters in the same way as in [1], i.e., each transmitter's location obeys the uniform distribution over a 2 Km $\times$ 2 Km square and the location of each receiver is uniformly generated in a disc with center at its corresponding transmitter and radius 400 m, excluding a radius of 10 m; (original) channel gains are given by $g_{kj} = 1/d_{kj}^4$ $(\forall\ k,\ j \in \mathcal{K})$, where d_{kj} is the Euclidean distance from the link of transmitter j to the link of receiver k. Each link's SINR target is set to be $\gamma_k = 2$ dB $(\forall\ k \in \mathcal{K})$ and the noise power is set to be $\eta_k = -90$ dBm $(\forall\ k \in \mathcal{K})$.

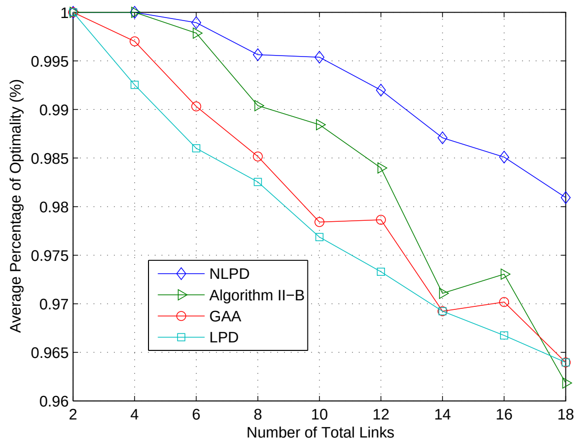

Fig. 1. The ratio of average number of supported links by different algorithms to the cardinality of the maximum admissible set versus the number of total links in small networks.

The power budget of the link of transmitter k is $p_k^{\max} = 2p_k^{\min}$ $(\forall\ k \in \mathcal{K})$, where $p_k^{\min}$ is the minimum power needed by link k to meet its SINR requirement without any interference from other links.

All figures are averaged over 200 Monte-Carlo runs. The parameter c_1 and c_2 in (23) are set to be 0.1 and 0.999 in all simulations[2]. The number of supported links, the total transmission power, and the execution time are the metrics we use for comparison. We compare the performance of the NLPD algorithm with that of two existing gradual removal algorithms[3]: the LPD algorithm in [1] and the Algorithm II-B in [2], since both of them have been reported to have close-to-optimal performance and perform much better than the GRN-DCPC algorithm in [3] in terms of the number of supported links [1], [2]. For completeness, we also compare the proposed algorithm with an existing gradual admission algorithm (GAA[4]) in [9]. When feasible, we use the global optimal solution obtained by "brute force" enumeration as benchmark.

[2]The numerical performance of the NLPD algorithm is not sensitive to the choice of c_1 and c_2 as long as $0 < c_1 \leq c_2 < 1$.

[3]Notice that the derived necessary conditions for all links in the network to be simultaneously supported can be used to accelerate other deflation algorithms (say the LPD algorithm in [1]). However, here we do not use the necessary conditions to accelerate these algorithms.

[4]In general, the heuristics based on gradual admission such as the GAA [9] are suitable to solve the problem in the strong interference channel where a low proportion of total links can be simultaneously supported, while the heuristics based on gradual removal such as the LPD algorithm [1] and the Algorithm II-B [2] are suitable to solve the problem in the low interference channel where a high proportion of total links can be simultaneously supported.

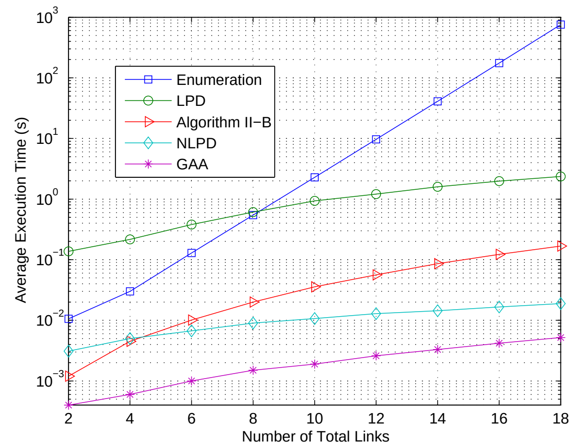

Fig. 2. Average execution time (in seconds) versus the number of total links in small networks.

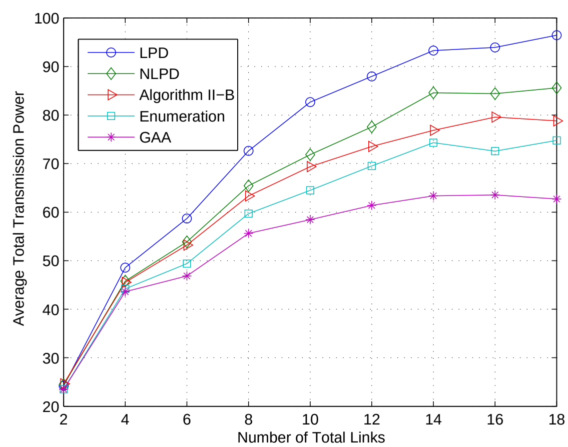

Fig. 3. Average total transmission power versus the number of total links in small networks.

Figs. 1 to 3 plot the performance comparison of various admission and power control algorithms for small networks. The vertical axis "Average Percentage of Optimality" in Fig. 1 shows the ratio of average number of supported links by different algorithms to the cardinality of the maximum admissible set. As depicted in Fig. 1, the NLPD algorithm can support the maximum number of links when $K = 2$ and 4; however, it does not always find the maximum admissible set with minimum total transmission power. This can be observed from Fig. 3 since it requires more total transmission power than the global minimum found by enumeration. Fig. 1 shows that the proposed NLPD algorithm supports the largest number of links among all the tested algorithms. In fact, compared to the LPD algorithm, the proposed NLPD

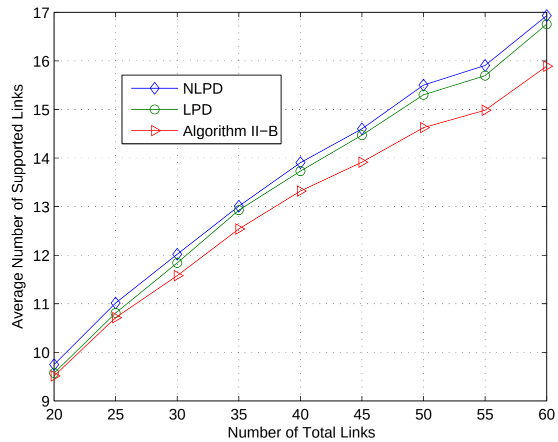

Fig. 4. Average number of supported links versus the number of total links in large networks.

algorithm can support more links with less total transmission power, and does so with substantially less CPU time in small networks.

The comparison of the GAA and the ENU algorithm in Figs. 1 and 3 reveals the fact that a small difference of the number of supported links may lead to a large difference of the total transmission power[5]. For instance, when the number of total links is 18, the difference of average number of supported links between the ENU algorithm and the GAA is $9.4350 - 9.0950 = 0.3400$, while the difference of average total transmission power between these two algorithms is $74.7772 - 62.7146 = 12.0626$. This fact explains why the NLPD algorithm transmits more power than the Algorithm II-B and the GAA, since the NLPD algorithm supports (despite a little) more links than the two algorithms.

In particular, Fig. 2 shows that the GAA performs the best in terms of the CPU time, and the NLPD algorithm ranks the second, which is slightly slower than the GAA. We remark that the CPU time comparison of these two algorithms depends on the simulation scenario. Since the NLPD algorithm gradually removes links from the network until all the remaining links can be simultaneously supported and the GAA gradually admits links until no link can be simultaneously supported with the already admitted links, we expect that the NLPD algorithm will take less CPU time than the GAA in the weak interference channel where a high proportion of total links in the network can be simultaneously supportable.

The comparison of the NLPD algorithm, the LPD algorithm, and the Algorithm II-B in large networks

[5]Notice that the GAA always transmits less power than the ENU algorithm.

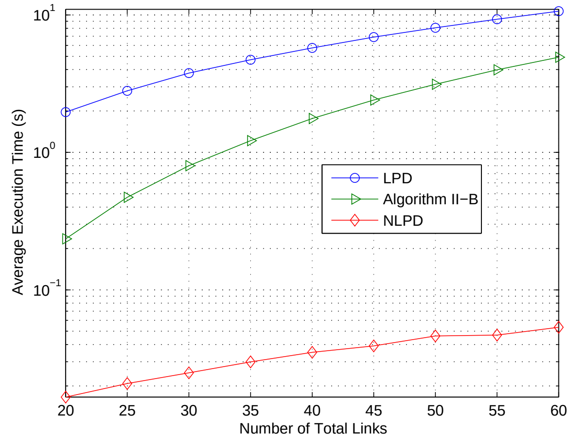

Fig. 5. Average execution time (in seconds) versus the number of total links in large networks.

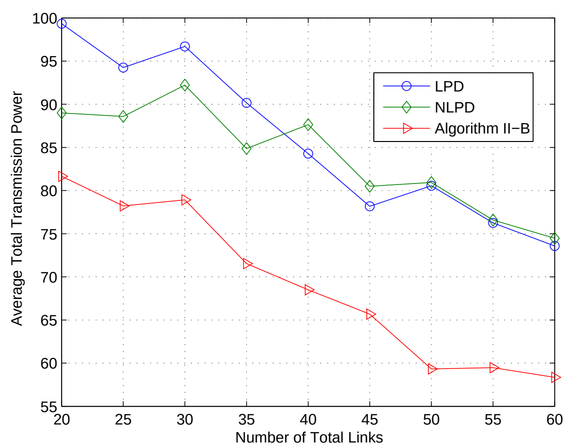

Fig. 6. Average total transmission power versus the number of total links in large networks.

is illustrated as Figs. 4 to 6. We can see from them that the NLPD algorithm supports more links and at the same time takes significantly less execution time than the LPD algorithm and the Algorithm II-B.

In a nutshell, the performance of the NLPD algorithm is better than that of the other two gradual removal algorithms (the Algorithm II-B and the LPD algorithm) in terms of the number of supported links and the execution time, while the Algorithm II-B transmits the least power among the tested algorithms. The reason why the NLPD algorithm transmits more power than the Algorithm II-B is because it supports more links than the Algorithm II-B. When compared to the LPD algorithm, the proposed NLPD algorithm can not only use less (or nearly equal) total power to support more links, but also has a significant reduction in CPU time. This performance improvement is a result of the new LP reformulation (14), the new removal strategy (25), and the necessary condition (30). In particular, the new LP approximation (14) is the key to the increase in the number of supported links and the save of the total transmission power. As we have mentioned before, although both the LP approximation (14) and the LP approximation (15) used in the NLPD algorithm and the LPD algorithm can be viewed as weighted ℓ_1-relaxations of the sparse optimization problem, the choice of the weight vector plays the key role in the approximation performance. Due to the introduction of the scaled channel, our choice of the weight vector is natural. Moreover, we offer a choice of the parameter α, which combines the admission control term with the power control term. Our choice of the parameter α not only exactly characterizes the original joint power and admission control problem, but also ensures the corresponding linear programming relaxation never removes a link unnecessarily. The significant reduction in the CPU time is because of the use of the necessary condition (30). We can iteratively remove the strong interfering links from the network until the necessary condition holds true, and therefore accelerate the deflation process significantly.

V. Conclusions

In this paper, we consider the NP-hard problem of joint power and admission control and reformulate it as a sparse optimization problem based on the balancing lemma. We propose a new linear programming relaxation for this problem and derive two easy-to-check necessary conditions for all links in the network to be simultaneously supportable. The new LP relaxation and the two necessary conditions can be used to guide an iterative link removal procedure (the NLPD algorithm), resulting in an efficient and effective solution for the joint power and admission control problem. We remark that the proposed NLPD algorithm can be easily extended to the cognitive underlay networks.

Appendix A
Proof of Theorem 1

We first establish the equivalence between problem (1) and the intermediate problem (9) in the sense that the optimal value of problem (1) is M if and only if the minimum value of problem (9) is $K-M$. Let us first show the "only if" direction. Suppose a set $\mathcal{I}\subseteq\mathcal{K}$ of M links can be supported in (1), so $\text{SINR}_k\geq\gamma_k$ for some power allocation $\tilde{\mathbf{p}}$ for all $k\in\mathcal{I}$. Notice that this is equivalent to the existence of $\mathbf{0}\leq\tilde{\mathbf{q}}\leq\mathbf{e}$ such that $\mathbf{A}_{\mathcal{I}}\tilde{\mathbf{q}}-\mathbf{c}_{\mathcal{I}}\geq\mathbf{0}$. Consider the linear subsystem in the sub-vector $\mathbf{q}_{\mathcal{I}}$

$$\mathbf{A}_{\mathcal{I}\mathcal{I}}\mathbf{q}_{\mathcal{I}}-(\mathbf{c}_{\mathcal{I}}-\mathbf{A}_{\mathcal{I}\mathcal{I}^c}\tilde{\mathbf{q}}_{\mathcal{I}^c})\geq\mathbf{0}, \tag{32}$$

where $\mathcal{I}^c$ denotes the complement of $\mathcal{I}$ with respect to $\mathcal{K}$. Since

$$\mathbf{A}_{\mathcal{I}\mathcal{I}}\tilde{\mathbf{q}}_{\mathcal{I}} - (\mathbf{c}_{\mathcal{I}} - \mathbf{A}_{\mathcal{I}\mathcal{I}^c}\tilde{\mathbf{q}}_{\mathcal{I}^c}) = \mathbf{A}_{\mathcal{I}}\tilde{\mathbf{q}} - \mathbf{c}_{\mathcal{I}} \geq \mathbf{0},$$

it follows that $\mathbf{q}_{\mathcal{I}} = \tilde{\mathbf{q}}_{\mathcal{I}} \geq \mathbf{0}$ is a feasible solution of (32). Moreover, notice that the off-diagonal entries of $\mathbf{A}$ are nonpositive, so we have $\mathbf{A}_{\mathcal{I}\mathcal{I}^c} \leq \mathbf{0}$, implying $\mathbf{c}_{\mathcal{I}} - \mathbf{A}_{\mathcal{I}\mathcal{I}^c}\tilde{\mathbf{q}}_{\mathcal{I}^c} \geq \mathbf{c}_{\mathcal{I}} > \mathbf{0}$. Since the submatrix $\mathbf{A}_{\mathcal{I}\mathcal{I}}$ still satisfies the assumptions of Lemma 1, we can invoke Lemma 1 to the linear subsystem (32) to deduce the existence of a feasible $\mathbf{c}_{\mathcal{I}} \leq \bar{\mathbf{q}}_{\mathcal{I}} \leq \tilde{\mathbf{q}}_{\mathcal{I}}$ such that

$$\mathbf{A}_{\mathcal{I}\mathcal{I}}\bar{\mathbf{q}}_{\mathcal{I}} - \mathbf{c}_{\mathcal{I}} + \mathbf{A}_{\mathcal{I}\mathcal{I}^c}\tilde{\mathbf{q}}_{\mathcal{I}^c} = \mathbf{0}.$$

Define $\bar{\mathbf{q}}_{\mathcal{I}^c} = \tilde{\mathbf{q}}_{\mathcal{I}^c}$ and let $\bar{\mathbf{x}} = \mathbf{A}\bar{\mathbf{q}} - \mathbf{c}$. Then the above equality means $\bar{\mathbf{x}}_{\mathcal{I}} = \mathbf{0}$, which further implies that at most $K - M$ components of $\bar{\mathbf{x}}$ are nonzero. Hence the optimal value of problem (9) is at most $K - M$.

To show the "if" direction, suppose that the optimal value of (9) is $K - M$ and $(\mathbf{x}^*, \mathbf{q}^*)$ is an optimal solution. Then $\|\mathbf{x}^*\|_0 = K - M$, implying $\mathbf{x}^*_{\mathcal{I}} = \mathbf{0}$ for some index set $\mathcal{I} \subseteq \mathcal{K}$ with $|\mathcal{I}| = M$. Since $\mathbf{x}^*_{\mathcal{I}} = \mathbf{A}_{\mathcal{I}}\mathbf{q}^* - \mathbf{c}_{\mathcal{I}} = \mathbf{0}$, it follows from the definition of $\mathbf{A}$ and $\mathbf{c}$ that

$$\text{SINR}_k = \gamma_k, \quad \forall\, k \in \mathcal{I}.$$

Thus, all links in $\mathcal{I}$ are supported at their target SINR levels, implying that the optimal value of (1) is at least M. This establishes the equivalence between (1) and (9).

We now establish the equivalence between problems (7) and (9) (in the sense of supporting the maximum number of links). We claim that the optimal value of (9) is M if and only if the optimal value of (7) is in the interval $(M, M+1)$. We argue the "only if" and "if" directions separately. Recall the fact $0 < \alpha < \alpha_1 = 1/(\mathbf{p}^{\max})^T\mathbf{e}$ (cf. (8)) which implies

$$\alpha\, (\mathbf{p}^{\max})^T\mathbf{q} < 1, \quad \text{for any } \mathbf{0} \leq \mathbf{q} \leq \mathbf{e}.$$

Consequently, the total contribution from the second term in the objective function of (7) cannot exceed 1, regardless of the power allocation $\mathbf{q}$. This immediately shows that if the optimal value of (9) is M, then the optimal value of (7) must be in the interval $(M, M+1)$. To argue the converse, we note that the function $\|\mathbf{x}\|_0$ is discontinuous with an increment of 1, implying that

$$\|\mathbf{x}\|_0 + \alpha\, (\mathbf{p}^{\max})^T\mathbf{q} \geq \|\mathbf{x}\|_0 \geq M + 1 > M + \alpha\, (\mathbf{p}^{\max})^T\mathbf{q}^*,$$

for any choice of $\mathbf{0} \leq \mathbf{q} \leq \mathbf{e}$, as long as $\|\mathbf{x}\|_0 > M$. Thus, the global minimum of (7) must be achieved at a power allocation for which $\|\mathbf{x}\|_0 = M$ holds, i.e., $\|\mathbf{x}\|_0$ is fully minimized by (9). This establishes the equivalence between (9) and (7).

Finally, if there are multiple sets of M links that are simultaneously supportable, then they all induce the same objective value in the first term of the objective function in (7). In this case, the second term (i.e., $(\mathbf{p}^{\max})^T \mathbf{q}$) will play the role to select the one set of M links which requires the least amount of total transmission power. In other words, $(\mathbf{p}^{\max})^T \mathbf{q}^* = \mathbf{e}^T \mathbf{p}^*$ is the minimum total power required to support any M links in the network.

Appendix B
Proof of Theorem 2

Notice that the problem (14) is equivalent to

$$\begin{aligned} \min_{\mathbf{x},\,\mathbf{q}} \quad & \|\mathbf{x}\|_1 + \alpha\,(\mathbf{p}^{\max})^T \mathbf{q} \\ \text{s.t.} \quad & \mathbf{x} = \mathbf{A}\mathbf{q} - \mathbf{c},\ \mathbf{x} \le \mathbf{0}, \\ & \mathbf{0} \le \mathbf{q} \le \mathbf{e}. \end{aligned}$$

Thus, to show the equivalence of (14) and (13) we only need to show that any optimal solution $(\tilde{\mathbf{x}}, \tilde{\mathbf{q}})$ of (13) always satisfies $\tilde{\mathbf{x}} = \mathbf{A}\tilde{\mathbf{q}} - \mathbf{c} \le \mathbf{0}$.

Denote $\mathcal{K}^{+} = \{\, k \mid \tilde{x}_k > 0 \,\}$, $\mathcal{K}^{=} = \{\, k \mid \tilde{x}_k = 0 \,\}$, and $\mathcal{K}^{-} = \{\, k \mid \tilde{x}_k < 0 \,\}$. We claim $|\mathcal{K}^{+}| = 0$, or equivalently $\tilde{\mathbf{x}} \le \mathbf{0}$. Assume the contrary so that $|\mathcal{K}^{+}| \ge 1$. We will derive a contradiction. Let $\mathcal{I} = \mathcal{K}^{+} \cup \mathcal{K}^{=}$. Then $\tilde{\mathbf{x}}_{\mathcal{I}} = \mathbf{A}_{\mathcal{I}}\tilde{\mathbf{q}} - \mathbf{c}_{\mathcal{I}} \ge \mathbf{0}$, which further implies

$$\mathbf{A}_{\mathcal{I}\mathcal{I}}\tilde{\mathbf{q}}_{\mathcal{I}} - (\mathbf{c}_{\mathcal{I}} - \mathbf{A}_{\mathcal{I}\mathcal{I}^c}\tilde{\mathbf{q}}_{\mathcal{I}^c}) \ge \mathbf{0}.$$

Hence, $\tilde{\mathbf{q}}_{\mathcal{I}}$ is a feasible solution to the following linear subsystem in $\mathbf{q}_{\mathcal{I}}$:

$$\mathbf{A}_{\mathcal{I}\mathcal{I}}\mathbf{q}_{\mathcal{I}} - (\mathbf{c}_{\mathcal{I}} - \mathbf{A}_{\mathcal{I}\mathcal{I}^c}\tilde{\mathbf{q}}_{\mathcal{I}^c}) \ge \mathbf{0}. \tag{33}$$

Since $\mathbf{A}_{\mathcal{I}\mathcal{I}^c} \le \mathbf{0}$ (the off-diagonals of $\mathbf{A}$ are nonpositive), it follows that $\mathbf{c}_{\mathcal{I}} - \mathbf{A}_{\mathcal{I}\mathcal{I}^c}\tilde{\mathbf{q}}_{\mathcal{I}^c} \ge \mathbf{c}_{\mathcal{I}} > \mathbf{0}$. It can be checked that the other assumptions of Lemma 1 all hold for (33) so that there exists a vector $\bar{\mathbf{q}}_{\mathcal{I}}$ such that $\mathbf{c}_{\mathcal{I}} \le \bar{\mathbf{q}}_{\mathcal{I}} \le \tilde{\mathbf{q}}_{\mathcal{I}}$ and $\mathbf{A}_{\mathcal{I}\mathcal{I}}\bar{\mathbf{q}}_{\mathcal{I}} - (\mathbf{c}_{\mathcal{I}} - \mathbf{A}_{\mathcal{I}\mathcal{I}^c}\tilde{\mathbf{q}}_{\mathcal{I}^c}) = \mathbf{0}$. Define $\bar{\mathbf{q}}_{\mathcal{I}^c} = \tilde{\mathbf{q}}_{\mathcal{I}^c}$. Then we have

$$\mathbf{A}_{\mathcal{I}}\bar{\mathbf{q}} - \mathbf{c}_{\mathcal{I}} = \mathbf{A}_{\mathcal{I}\mathcal{I}}\bar{\mathbf{q}}_{\mathcal{I}} - (\mathbf{c}_{\mathcal{I}} - \mathbf{A}_{\mathcal{I}\mathcal{I}^c}\tilde{\mathbf{q}}_{\mathcal{I}^c}) = \mathbf{0}. \tag{34}$$

Moreover, we have $\mathbf{0} \le \bar{\mathbf{q}} \le \tilde{\mathbf{q}} \le \mathbf{e}$, so $\bar{\mathbf{q}}$ is a feasible power allocation. With this new power allocation $\bar{\mathbf{q}}$, there holds

$$\mathbf{A}_{\mathcal{I}^c}\bar{\mathbf{q}} - \mathbf{c}_{\mathcal{I}^c} = \mathbf{A}_{\mathcal{I}^c\mathcal{I}^c}\tilde{\mathbf{q}}_{\mathcal{I}^c} + \mathbf{A}_{\mathcal{I}^c\mathcal{I}}\bar{\mathbf{q}}_{\mathcal{I}} - \mathbf{c}_{\mathcal{I}^c} \ge \mathbf{A}_{\mathcal{I}^c\mathcal{I}^c}\tilde{\mathbf{q}}_{\mathcal{I}^c} + \mathbf{A}_{\mathcal{I}^c\mathcal{I}}\tilde{\mathbf{q}}_{\mathcal{I}} - \mathbf{c}_{\mathcal{I}^c} = \mathbf{A}_{\mathcal{I}^c}\tilde{\mathbf{q}} - \mathbf{c}_{\mathcal{I}^c},$$

where the inequality follows from $\mathbf{A}_{\mathcal{I}^c\mathcal{I}} \le \mathbf{0}$ and $\bar{\mathbf{q}} \le \tilde{\mathbf{q}}$. This further implies

$$(\mathbf{c}_{\mathcal{I}^c} - \mathbf{A}_{\mathcal{I}^c}\bar{\mathbf{q}})_+ \le (\mathbf{c}_{\mathcal{I}^c} - \mathbf{A}_{\mathcal{I}^c}\tilde{\mathbf{q}})_+$$

where $(\cdot)_+$ denotes the projection to the nonnegative orthant. Since $\mathbf{c}_{\mathcal{I}} - \mathbf{A}_{\mathcal{I}}\bar{\mathbf{q}} = \mathbf{0}$ (cf. (34)), it follows that

$$(\mathbf{c} - \mathbf{A}\bar{\mathbf{q}})_+ \leq (\mathbf{c} - \mathbf{A}\tilde{\mathbf{q}})_+ \tag{35}$$

with the inequality holds true *strictly* for entries indexed by $\mathcal{K}^+$. Define a potential function

$$p(\mathbf{q}) = \mathbf{e}^T(\mathbf{c} - \mathbf{A}\mathbf{q})_+ + \alpha\ (\mathbf{p}^{\max})^T\,\mathbf{q}.$$

Clearly, we have

$$p(\mathbf{q}) \leq \|\mathbf{A}\mathbf{q} - \mathbf{c}\|_1 + \alpha\ (\mathbf{p}^{\max})^T\,\mathbf{q}, \quad \forall\, \mathbf{q} \geq \mathbf{0},$$

where the equality holds whenever $\mathbf{A}\mathbf{q} - \mathbf{c} \leq \mathbf{0}$. Since $\mathbf{0} \leq \bar{\mathbf{q}} \leq \tilde{\mathbf{q}}$, we can use (35) to obtain

$$p(\bar{\mathbf{q}}) = \mathbf{e}^T(\mathbf{c} - \mathbf{A}\bar{\mathbf{q}})_+ + \alpha\ (\mathbf{p}^{\max})^T\,\bar{\mathbf{q}} < \mathbf{e}^T(\mathbf{c} - \mathbf{A}\tilde{\mathbf{q}})_+ + \alpha\ (\mathbf{p}^{\max})^T\,\tilde{\mathbf{q}} = p(\tilde{\mathbf{q}}).$$

Moreover, it follows from $|\mathcal{K}^+| \geq 1$ that

$$\|\mathbf{A}\bar{\mathbf{q}} - \mathbf{c}\|_0 = K - |\mathcal{I}| = K - |\mathcal{K}^=| - |\mathcal{K}^+| < K - |\mathcal{K}^=| = \|\mathbf{A}\tilde{\mathbf{q}} - \mathbf{c}\|_0.$$

We claim that, without loss of generality, we can assume $\mathbf{A}_{\mathcal{I}^c}\bar{\mathbf{q}} - \mathbf{c}_{\mathcal{I}^c} \leq \mathbf{0}$ so that $\mathbf{A}\bar{\mathbf{q}} - \mathbf{c} \leq \mathbf{0}$. This is because otherwise we can repeat the above steps by replacing the vector $\tilde{\mathbf{q}}$ with $\bar{\mathbf{q}}$ to find a new vector $\mathbf{0} \leq \hat{\mathbf{q}} \leq \bar{\mathbf{q}}$ such that

$$p(\hat{\mathbf{q}}) < p(\bar{\mathbf{q}}) < p(\tilde{\mathbf{q}}) \quad \text{and} \quad \|\mathbf{A}\hat{\mathbf{q}} - \mathbf{c}\|_0 < \|\mathbf{A}\bar{\mathbf{q}} - \mathbf{c}\|_0 < \|\mathbf{A}\tilde{\mathbf{q}} - \mathbf{c}\|_0.$$

Since the ℓ_0-norm can be reduced at most finitely many times, it follows that by repeatedly applying the above steps we will eventually obtain a power allocation (still denote by $\bar{\mathbf{q}}$) such that

$$\mathbf{0} \leq \bar{\mathbf{q}} \leq \tilde{\mathbf{q}} \leq \mathbf{e}, \quad p(\bar{\mathbf{q}}) < p(\tilde{\mathbf{q}}), \quad \mathbf{A}\bar{\mathbf{q}} - \mathbf{c} \leq \mathbf{0}.$$

Notice that when $\mathbf{A}\bar{\mathbf{q}} - \mathbf{c} \leq \mathbf{0}$ we have

$$\|\mathbf{A}\bar{\mathbf{q}} - \mathbf{c}\|_1 + \alpha\ (\mathbf{p}^{\max})^T\,\bar{\mathbf{q}} = p(\bar{\mathbf{q}}) < p(\tilde{\mathbf{q}}) \leq \|\mathbf{A}\tilde{\mathbf{q}} - \mathbf{c}\|_1 + \alpha\ (\mathbf{p}^{\max})^T\,\tilde{\mathbf{q}}.$$

This contradicts the optimality of $(\tilde{\mathbf{x}}, \tilde{\mathbf{q}})$. The proof is complete.

Appendix C
Proof of Proposition 1

Let us first show that $\mathbf{q}^*$ satisfying (27) solves the LP (14). The KKT condition of problem (14) can be written as:

$$\begin{cases} -\mathbf{A}^T\mathbf{e} + \alpha\mathbf{p}^{\max} + \mathbf{A}^T\boldsymbol{\lambda} - \boldsymbol{\xi} + \boldsymbol{\eta} = \mathbf{0}, \\ \boldsymbol{\lambda}^T(\mathbf{c} - \mathbf{A}\mathbf{q}) = 0,\ \boldsymbol{\xi}^T\mathbf{q} = 0,\ \boldsymbol{\eta}^T(\mathbf{e} - \mathbf{q}) = 0, \\ \boldsymbol{\lambda} \geq \mathbf{0},\ \boldsymbol{\xi} \geq \mathbf{0},\ \boldsymbol{\eta} \geq \mathbf{0}, \\ \mathbf{c} - \mathbf{A}\mathbf{q} \geq \mathbf{0},\ \mathbf{q} \geq \mathbf{0},\ \mathbf{e} - \mathbf{q} \geq \mathbf{0}, \end{cases} \tag{36}$$

where $\boldsymbol{\lambda}$, $\boldsymbol{\xi}$, and $\boldsymbol{\eta}$ are the Lagrangian variables corresponding to the constraints $\mathbf{c} - \mathbf{A}\mathbf{q} \geq \mathbf{0}$, $\mathbf{q} \geq \mathbf{0}$, and $\mathbf{e} \geq \mathbf{q}$, respectively. Thus, to prove $\mathbf{q}^*$ solves the LP (14), we need to prove that there are nonnegative vectors

$$\boldsymbol{\lambda} = \begin{pmatrix} \boldsymbol{\lambda}_1 \\ \boldsymbol{\lambda}_2 \end{pmatrix},\ \boldsymbol{\xi} = \begin{pmatrix} \boldsymbol{\xi}_1 \\ \boldsymbol{\xi}_2 \end{pmatrix},\ \boldsymbol{\eta} = \begin{pmatrix} \boldsymbol{\eta}_1 \\ \boldsymbol{\eta}_2 \end{pmatrix}$$

such that when the conditions in (28) hold, then $(\boldsymbol{\lambda},\ \boldsymbol{\xi},\ \boldsymbol{\eta},\ \mathbf{q}^*)$ will satisfy all the conditions in (36). Recalling (27) and setting $\boldsymbol{\eta} = \mathbf{0},\ \boldsymbol{\xi}_1 = \mathbf{0}$, and $\boldsymbol{\lambda}_2 = \mathbf{0}$, the conditions in (36) reduce to

$$\begin{cases} -\mathbf{A}_{11}^T\mathbf{e} - \mathbf{A}_{21}^T\mathbf{e} + \alpha\mathbf{p}_1^{\max} + \mathbf{A}_{11}^T\boldsymbol{\lambda}_1 = \mathbf{0}, \\ -\mathbf{A}_{12}^T\mathbf{e} - \mathbf{A}_{22}^T\mathbf{e} + \alpha\mathbf{p}_2^{\max} + \mathbf{A}_{12}^T\boldsymbol{\lambda}_1 - \boldsymbol{\xi}_2 = \mathbf{0}. \end{cases} \tag{37}$$

Since the conditions in (28) are satisfied, it follows that

$$\begin{cases} \boldsymbol{\lambda}_1 = \mathbf{e} + \left(\mathbf{A}_{11}^T\right)^{-1}\mathbf{A}_{21}^T\mathbf{e} - \alpha\left(\mathbf{A}_{11}^T\right)^{-1}\mathbf{p}_1^{\max} \geq \mathbf{0}, \\ \boldsymbol{\xi}_2 = \mathbf{A}_{12}^T\left(\mathbf{A}_{11}^T\right)^{-1}\mathbf{A}_{21}^T\mathbf{e} - \mathbf{A}_{22}^T\mathbf{e} - \alpha\mathbf{A}_{12}^T\left(\mathbf{A}_{11}^T\right)^{-1}\mathbf{p}_1^{\max} + \alpha\mathbf{p}_2^{\max} \geq \mathbf{0}. \end{cases} \tag{38}$$

Hence $\mathbf{q}^*$ is a solution to the LP (14).

We now show that $\mathbf{q}^*$ is the unique solution to the LP (14). According to [28], a solution to the LP $\mathcal{P}$ is unique if and only if it remains a solution to all LPs obtained from $\mathcal{P}$ by arbitrary but sufficiently small perturbation of its cost vector. Thus, to show the uniqueness of $\mathbf{q}^*$, it is sufficient to show that for each $\mathbf{d}$, there exists an $\epsilon > 0$ such that $\mathbf{q}^*$ remains a solution to the perturbed LP

$$\begin{aligned} \min_{\mathbf{q}} \quad & \mathbf{e}^T(\mathbf{c} - \mathbf{A}\mathbf{q}) + \alpha\left(\mathbf{p}^{\max}\right)^T\mathbf{q} + \epsilon\mathbf{d}^T\mathbf{q} \\ \text{s.t.} \quad & \mathbf{c} - \mathbf{A}\mathbf{q} \geq \mathbf{0}, \\ & \mathbf{0} \leq \mathbf{q} \leq \mathbf{e}. \end{aligned} \tag{39}$$

Similar to the first part, we can write the KKT condition of problem (39), and check that for any perturbed vector $\mathbf{d}$, there exists $\epsilon > 0$ such that $\mathbf{q}^*$ is still a solution to the LP (39) if the conditions in (28) are satisfied. This completes the proof of Proposition 1.

ACKNOWLEDGMENT

The authors wish to thank N. D. Sidiropoulos of University of Minnesota for his help in numerical simulations. The authors also would like to thank the anonymous reviewers for their useful comments.

REFERENCES

[1] I. Mitliagkas, N. D. Sidiropoulos, and A. Swami, "Joint power and admission control for ad-hoc and cognitive underlay networks: Convex approximation and distributed implementation," *IEEE Trans. Wireless Commun.*, vol. 10, no. 12, pp. 4110–4121, Dec. 2011.

[2] H. Mahdavi-Doost, M. Ebrahimi, and A. K. Khandani, "Characterization of SINR region for interfering links with constrained power," *IEEE Trans. Inf. Theory*, vol. 56, no. 6, pp. 2816–2828, Jun. 2010.

[3] M. Andersin, Z. Rosberg, and J. Zander, "Gradual removals in cellular PCS with constrained power control and noise," *Wireless Netw.*, vol. 2, no. 1, pp. 27–43, Mar. 1996.

[4] G. J. Foschini and Z. Miljanic, "A simple distributed autonomous power control algorithm and its convergence," *IEEE Trans. Veh. Technol.*, vol. 42, no. 4, pp. 641–646, Nov. 1993.

[5] N. Bambos, S.-C. Chen, and G. J. Pottie, "Channel access algorithms with active link protection for wireless communication networks with power control," *IEEE Trans. Netw.*, vol. 8, no. 5, pp. 583–597, Oct. 2000.

[6] R. D. Yates, "A framework for uplink power control in cellular radio systems," *IEEE J. Sel. Areas Commun.*, vol. 13, no. 7, pp. 1341–1347, Sept. 1995.

[7] S. A. Grandhi, J. Zander, and R. Yates, "Constrained power control," *Wireless Personal Commun.*, vol. 1, no. 4, pp. 257–270, Dec. 1994.

[8] M. Andersin, Z. Rosberg, and J. Zander, "Soft and safe admission control in cellular networks," *IEEE/ACM Trans. Netw.*, vol. 5, no. 2, pp. 255–265, Apr. 1997.

[9] D. I. Evangelinakis, N. D. Sidiropoulos, and A. Swami, "Joint admission and power control using branch & bound and gradual admissions," *Proc. Int. Workshop Signal Process. Advances Wireless Commun.,* Jun. 2010, pp. 1–5.

[10] J. Zander, "Performance of optimum transmitter power control in cellular radio systems," *IEEE Trans. Veh. Technol.*, vol. 41, no. 1, pp. 57–62, Feb. 1992.

[11] J. Zander, "Distributed cochannel interference control in cellular radio systems," *IEEE Trans. Veh. Technol.*, vol. 41, no. 3, pp. 305–311, Aug. 1992.

[12] T.-H. Lee, J.-C. Lin, and Y.-T. Su, "Downlink power control algorithms for cellular radio systems," *IEEE Trans. Veh. Technol.*, vol. 44, no. 1, pp. 89–94, Feb. 1995.

[13] T.-H. Lee and J.-C. Lin, "A fully distributed power control algorithm for cellular mobile systems," *IEEE J. Sel. Areas Commun.*, vol. 14, no. 4, pp. 692–697, May 1996.

[14] J.-T. Wang and T.-H. Lee, "Non-reinitialized fully distributed power control algorithm," *IEEE Commun. Lett.*, vol. 3, no. 12, pp. 329–331, Dec. 1999.

[15] E. Matskani, N. D. Sidiropoulos, Z.-Q. Luo, and L. Tassiulas, "Convex approximation techniques for joint multiuser downlink beamforming and admission control," *IEEE Trans. Wireless Commun.*, vol. 7, no. 7, pp. 2682–2693, Jul. 2008.

[16] M. H. Ahmed, "Call admission control in wireless networks: A comprehensive survey," *IEEE Commun. Surveys*, vol. 7, no. 1, pp. 50–69, First Quarter, 2005.

[17] A. Behzad, I. Rubin, and P. Chakravarty, "Optimum integrated link scheduling and power control for ad hoc wireless networks," *Proc. IEEE Int. Conf. Wireless and Mobile Computing, Netw., and Commun.,* vol. 3, Aug. 2005, pp. 275–283.

[18] J.-T. Wang, "Admission control with distributed joint diversity and power control for wireless networks," *IEEE Trans. Veh. Technol.*, vol. 58, no.1, pp. 409–419, Jan. 2009.

[19] T. Elbatt and A. Ephremides, "Joint scheduling and power control for wireless ad hoc networks," *IEEE Trans. Wireless Commun.*, vol. 3, no. 1, pp. 74–85, Jan. 2004.

[20] Y.-F. Liu, Y.-H. Dai, and Z.-Q. Luo, "Joint power and admission control via linear programming deflation," *Proceeding of ICASSP,* Mar. 2012, pp. 2873–2876.

[21] T. Holliday, A. Goldsmith, P. Glynn, and N. Bambos, "Distributed power and admission control for time varying wireless networks," *Proc. Global Telecommun. Conf.,* Nov. 2004, pp. 768–774.

[22] J. M. Aein, "Power balancing in systems employing frequency reuse," *COMSAT Tech. Rev.,* vol. 3, no. 2, Fall 1973.

[23] R. W. Nettleton and H. Alavi, "Power control for a spread-spectrum cellular mobile radio system", *Proc. IEEE Veh. Technol. Conf.,* May 1983, pp. 242–246.

[24] L. Trevisan, "Inapproximability of combinatorial optimization problems," *Technical Report TR04-065, Electronic Colloquium on Computational Complexity*, Jul. 2004.

[25] Y. Saad, *Iterative Methods for Sparse Linear Systems,* 2nd ed. New York, U.S.A.: PSW publishing, Jan. 2000.

[26] A. Ben-Tal and A. Nemirovski, *Lectures on Modern Convex Optimization.* Philadelphia, U.S.A.: SIAM-MPS Series on Optimization, SIAM Publications, 2001.

[27] G. H. Golub and C. F. Van Loan, *Matrix Computations*, 3rd ed. Baltimore, MD, U.S.A.: The Johns Hopkins Press, 1996.

[28] O. L. Mangasarian, "Uniqueness of solution in linear programming," *Computer Sciences Technical Report #316,* Feb. 1978.

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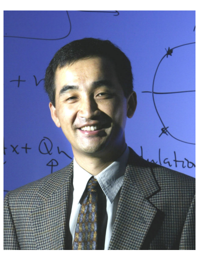

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