Max-Min Fairness Linear Transceiver Design Problem for a Multi-User SIMO Interference Channel Is Polynomial Time Solvable

Ya-Feng Liu, Mingyi Hong, and Yu-Hong Dai

Abstract

Consider the linear transceiver design problem for a multi-user single-input multi-output (SIMO) interference channel. Assuming perfect channel knowledge, we formulate this problem as one of maximizing the minimum signal to interference plus noise ratio (SINR) among all the users, subject to individual power constraints at each transmitter. We prove in this letter that the max-min fairness linear transceiver design problem for the SIMO interference channel can be solved to global optimality in polynomial time. We further propose a low-complexity inexact cyclic coordinate ascent algorithm (ICCAA) to solve this problem. Numerical simulations show the proposed algorithm can efficiently find the global optimal solution of the considered problem.

Index Terms

Beamforming, complexity, ICCAA, max-min fairness, SIMO interference channel.

I. Introduction

In a multi-user multi-input multi-output (MIMO) interference channel, a number of linearly interfering transmitters simultaneously send private data to their respective receivers. When the number of users in the interference channel is large, multi-user interference becomes a major performance limiting factor. An effective approach to mitigate the multi-user interference is to jointly optimize the physical layer transmit-receive beamforming strategies for all users.

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In this letter, we design beamforming strategies to maximize the min-rate utility (equivalent to the minimum SINR utility), which places the highest emphasis on the user fairness. This max-min fairness linear transceiver design has been studied extensively in [1]–[9]. Among them, the authors in [1], [2] proposed to approximate the optimum by minimizing the sum of equally weighted inverse signal to interference ratios (SIR). For the single receive antenna case, the authors of [3] further extended this approach by choosing suitable weight factors with which the weighted sum of inverse SIR maximization can achieve optimal max-min fairness. The reference [4] developed an iterative algorithm by switching the optimization between uplink and downlink channels based on the uplink-downlink duality theory [4], [10]. Polynomial time algorithms capable of achieving global optimality have been proposed in [5]–[7] for the power control and/or transmit beamforming design problems, again for the single receive antenna case. These results imply that the max-min SINR precoder design problem with *fixed receive beamformers* can be solved in polynomial time [5]. For the case where there are more than two antennas per transmitter and more than one antenna per receiver, it was shown in [8] that the corresponding max-min fairness linear transceiver design problem is strongly NP-hard. For the *single-user* multi-carrier MIMO channel, [9] proposed a unified framework for the joint design of Tx-Rx beamforming.

We prove in this letter that the max-min fairness linear transceiver design problem in an uplink wireless system, where each transmitter (the mobile user) is equipped with a single antenna and each receiver (the base station) is equipped with multiple antennas is polynomial time solvable. Due to the specific SIMO case considered, the complexity status of minimum SINR optimization problem does not follow from any of the existing complexity results [6]–[8]. We also propose a low-complexity inexact cyclic coordinate ascent algorithm (ICCAA) to solve this problem.

II. Problem Formulation

Consider a K-user SIMO interference channel where each transmitter is equipped with a single antenna and each receiver is equipped with L antennas, respectively. For the single-carrier channel, the received signal at receiver k is

$$\mathbf{y}_k = \mathbf{h}_{kk}\sqrt{p_k}s_k + \sum_{j\neq k}\mathbf{h}_{kj}\sqrt{p_j}s_j + \mathbf{z}_k,$$

where $\mathbf{h}_{kj} \in \mathbb{C}^{L\times 1}$ is the channel vector from transmitter j to receiver k, $p_k \geq 0$ is the transmitted power by transmitter k, $s_k \in \mathbb{C}$ is the symbol that transmitter k wishes to send to receiver k, and $\mathbf{z}_k \in \mathbb{C}^{L\times 1}$ is the additive white Gaussian noise (AWGN) with distribution $\mathcal{CN}(\mathbf{0}, \sigma_k^2\mathbf{I})$. Let $\mathbf{u}_k \in \mathbb{C}^{L\times 1}$ be the receive beamformer for receiver k and a linear reception strategy is assumed. Then, the k-th receiver filters its received signal to obtain

$$\hat{s}_k = \mathbf{u}_k^\dagger \mathbf{y}_k = \mathbf{u}_k^\dagger \mathbf{h}_{kk}\sqrt{p_k}s_k + \sum_{j\neq k}\mathbf{u}_k^\dagger \mathbf{h}_{kj}\sqrt{p_j}s_j + \mathbf{u}_k^\dagger \mathbf{z}_k.$$

Treating interference as noise, we can write the SINR of user k as

$$\text{SINR}_k = \frac{|\mathbf{u}_k^\dagger \mathbf{h}_{kk}|^2 p_k}{\sigma_k^2\|\mathbf{u}_k\|^2 + \sum_{j\neq k}|\mathbf{u}_k^\dagger \mathbf{h}_{kj}|^2 p_j}. \tag{1}$$

In this letter, we consider the max-min fairness linear transceiver design problem as follows:

$$\begin{aligned} \max_{\{\mathbf{u},\mathbf{p}\}} \quad & \min_{k\in\mathcal{K}}\{\text{SINR}_k\} \\ \text{s.t.} \quad & 0 \leq p_k \leq P_k,\ k\in\mathcal{K}, \end{aligned} \tag{2}$$

where P_k denotes the power budget of transmitter k, $\mathbf{p} := (p_1; p_2, ...; p_K)$, $\mathbf{u} := (\mathbf{u}_1; \mathbf{u}_2; ...; \mathbf{u}_K)$, and $\mathcal{K} := \{1, 2, ..., K\}$. Introducing an auxiliary variable

$$\text{SINR} = \min_{k\in\mathcal{K}}\{\text{SINR}_k\},$$

we can equivalently rewrite the minimum SINR maximization problem (2) as

$$\begin{aligned} \max_{\{\mathbf{u},\mathbf{p},\text{SINR}\}} \quad & \text{SINR} \\ \text{s.t.} \quad & \text{SINR} \leq \text{SINR}_k,\ 0 \leq p_k \leq P_k,\ k\in\mathcal{K}. \end{aligned} \tag{3}$$

III. Computational Complexity Analysis

In this section, we investigate the complexity status of the optimization problem (3) (equivalent to problem (2)). We show that, when each transmitter is equipped with a single antenna (SIMO case), the corresponding max-min fairness design problem (3) can be solved to global optimality in polynomial time via solving a series of semi-definite programs (SDP). This result is different from the polynomial time solvability of the problem in the MISO case [5], [7], where each step solves a second order cone program (SOCP), and also different from the NP-hardness of the one in the MIMO case [8].

Specifically, the optimal receive beamformer for problem (3) is the linear minimum mean square error (LMMSE) receive beamformer given by

$$\mathbf{u}_k = \mathbf{M}_k^{-1}(\mathbf{p})\mathbf{h}_{kk},\ \mathbf{M}_k(\mathbf{p}) = \sigma_k^2\mathbf{I} + \sum_{j\in\mathcal{K}}\mathbf{h}_{kj}\mathbf{h}_{kj}^\dagger p_j \succ \mathbf{0},\ k\in\mathcal{K}. \tag{4}$$

Substituting these optimal LMMSE receive beamformers $\mathbf{u}_k$ into (3) and letting $\xi = 1/\text{SINR}$ yields

$$\begin{aligned} \min_{\{\xi,\mathbf{p}\}} \quad & \xi \\ \text{s.t.} \quad & p_k(1+\xi)\mathbf{h}_{kk}^\dagger\mathbf{M}_k^{-1}(\mathbf{p})\mathbf{h}_{kk} \geq 1, \\ & 0 \leq p_k \leq P_k,\ k\in\mathcal{K}. \end{aligned} \tag{5}$$

Next, we show that checking the feasibility of problem (5) with fixed ξ can be performed in polynomial time. To this end, we *fix* ξ in the above program (5) and consider the following optimization problem

$$\begin{aligned} \min_{\{\mathbf{p}\}} \quad & \sum_{k\in\mathcal{K}} p_k \\ \text{s.t.} \quad & p_k(1+\xi)\mathbf{h}_{kk}^{\dagger}\mathbf{M}_k^{-1}(\mathbf{p})\mathbf{h}_{kk} \geq 1, \\ & 0 \leq p_k \leq P_k,\ k\in\mathcal{K}. \end{aligned} \tag{6}$$

Clearly, for any fixed ξ, this problem has a solution if and only if ξ is feasible for problem (5). To efficiently solve (6), we further introduce a problem closely related to it:

$$\begin{aligned} \max_{\{\mathbf{p}\}} \quad & \sum_{k\in\mathcal{K}} p_k \\ \text{s.t.} \quad & p_k(1+\xi)\mathbf{h}_{kk}^{\dagger}\mathbf{M}_k^{-1}(\mathbf{p})\mathbf{h}_{kk} \leq 1, \\ & 0 \leq p_k \leq P_k,\ k\in\mathcal{K}. \end{aligned} \tag{7}$$

The difference between (6) and (7) lies in minimization versus maximization and the reversed SINR inequality constraints. It can be observed that problem (7) is always feasible ($\mathbf{p}=\mathbf{0}$ is a certificate for the feasibility); but problem (6) may not be feasible. Hence, these two problems are not equivalent to each other in general. The following proposition indicates that they are equivalent as long as problem (6) is feasible.

Proposition 3.1: Suppose that problem (6) is feasible with some given ξ. Then the optima of problem (6) and (7) are attained *if and only if* all the SINR constraints are met with equality. Furthermore, the point that achieves all the SINR constraints with equality is unique.

To establish Proposition 3.1, we first introduce Lemma 3.1.

Lemma 3.1: Consider the univariate functions $f(\alpha)=\mathbf{h}^{\dagger}\left(\mathbf{A}+\alpha\mathbf{B}\right)^{-1}\mathbf{h}$ and $g(\alpha)=\alpha f(\alpha)$, where $\mathbf{h}\neq\mathbf{0}\in\mathbb{C}^{n\times 1}$, $\mathbf{A}\succ\mathbf{0}\in\mathbb{C}^{n\times n}$, $\mathbf{B}\succeq\mathbf{0}\in\mathbb{C}^{n\times n}$ and $\alpha\in[0,+\infty)$. Then $f(\alpha)$ is a decreasing function and $g(\alpha)$ is a *strictly* increasing function.

The key insight into proving Lemma 3.1 is that the matrices $\mathbf{A}$ and $\mathbf{B}$ can be made diagonal simultaneously [11] when the conditions in Lemma 3.1 hold true. Due to the page limit, we omit the proof.

By Lemma 3.1, we know that

$$g_k(\mathbf{p})=g_k(p_k;\mathbf{p}_{-k})=p_k(1+\xi)\mathbf{h}_{kk}^{\dagger}\mathbf{M}_k^{-1}(\mathbf{p})\mathbf{h}_{kk},\forall\ k\in\mathcal{K},$$

is strictly increasing with $p_k\geq 0$ and is decreasing with each component p_j of $\mathbf{p}_{-k}$, where $\mathbf{p}_{-k}$ is the reduced vector of $\mathbf{p}$ by deleting the k-th component. We are now ready to prove Proposition 3.1.

$$
\begin{aligned}
g_{k_0}(\tilde{\mathbf{p}}) &= g_{k_0}(\tilde{p}_{k_0};\tilde{\mathbf{p}}_{-k_0}) \\
&\overset{(a)}{\geq} g_{k_0}(\tilde{p}_{k_0};\boldsymbol{\alpha}_{-k_0}\circ\hat{\mathbf{p}}_{-k_0}) \quad \text{(from the fact } \alpha_k\hat{p}_k \geq \tilde{p}_k \text{ for all } k\in\mathcal{K}) \\
&\overset{(b)}{=} \alpha_{k_0}\hat{p}_{k_0}(1+\xi)\mathbf{h}_{k_0k_0}^{\dagger}\left(\sigma_{k_0}^2\mathbf{I}+\alpha_{k_0}\textstyle\sum_{j\in\mathcal{K}^+}\mathbf{h}_{k_0j}\mathbf{h}_{k_0j}^{\dagger}\hat{p}_j+\sum_{j\notin\mathcal{K}^+}\mathbf{h}_{k_0j}\mathbf{h}_{k_0j}^{\dagger}\hat{p}_j\right)^{-1}\mathbf{h}_{k_0k_0} \\
&\overset{(c)}{>} \hat{p}_{k_0}(1+\xi)\mathbf{h}_{k_0k_0}^{\dagger}\left(\sigma_{k_0}^2\mathbf{I}+\textstyle\sum_{j\in\mathcal{K}^+}\mathbf{h}_{k_0j}\mathbf{h}_{k_0j}^{\dagger}\hat{p}_j+\sum_{j\notin\mathcal{K}^+}\mathbf{h}_{k_0j}\mathbf{h}_{k_0j}^{\dagger}\hat{p}_j\right)^{-1}\mathbf{h}_{k_0k_0} \\
&= \hat{p}_{k_0}(1+\xi)\mathbf{h}_{k_0k_0}^{\dagger}\left(\sigma_{k_0}^2\mathbf{I}+\textstyle\sum_{j\in\mathcal{K}}\mathbf{h}_{k_0j}\mathbf{h}_{k_0j}^{\dagger}\hat{p}_j\right)^{-1}\mathbf{h}_{k_0k_0} \\
&= g_{k_0}(\hat{\mathbf{p}}) \quad \text{(from the definition of } g_k(\hat{\mathbf{p}})) \\
&= 1 \quad \text{(from (8))},
\end{aligned} \tag{10}
$$

Proof of Proposition 3.1: Suppose that problem (6) is feasible and $\hat{\mathbf{p}}$ is one of its optimal solutions, then all the SINR constraints of problem (6) at the point $\hat{\mathbf{p}}$ are active, i.e.,

$$g_k(\hat{\mathbf{p}}) = 1,\ \forall\ k\in\mathcal{K}. \tag{8}$$

Otherwise, we can appropriately decrease $\hat{p}_k$ associated with the strict SINR constraint and strictly decrease the objective without violating the SINR constraints (from the monotonicity of $g_j(\mathbf{p})$ for all $j\in\mathcal{K}$ with respect to p_k) and individual power constraints.

Next, we show by contradiction that any $\hat{\mathbf{p}}$ satisfying (8) is also a solution to problem (7). If there exists some point $\tilde{\mathbf{p}}\neq\hat{\mathbf{p}}$ such that it is feasible to problem (7), namely, $g_k(\tilde{\mathbf{p}})\leq 1$ for all $k\in\mathcal{K}$, and $\sum_{k\in\mathcal{K}}\tilde{p}_k\geq\sum_{k\in\mathcal{K}}\hat{p}_k$, we derive a contradiction. In fact, denote the nonempty set $\mathcal{K}^+=\{k\in\mathcal{K}\mid\tilde{p}_k/\hat{p}_k>1\}$ and $k_0=\arg\max_{k\in\mathcal{K}^+}\{\tilde{p}_k/\hat{p}_k\}$ [1]. Define the vector $\boldsymbol{\alpha}$ with its k-th component being

$$\alpha_k=\begin{cases}\tilde{p}_{k_0}/\hat{p}_{k_0}>1, & \text{if } k\in\mathcal{K}^+;\\ 1, & \text{if } k\notin\mathcal{K}^+.\end{cases} \tag{9}$$

By the definition of $\boldsymbol{\alpha}$, we know $\tilde{p}_{k_0}=\alpha_{k_0}\hat{p}_{k_0}$ and $\alpha_k\hat{p}_k\geq\tilde{p}_k$ for all $k\in\mathcal{K}$. Using the above definitions, we can express the function $g_{k_0}(\tilde{\mathbf{p}})$ as in (10), which contradicts the feasibility of $\tilde{\mathbf{p}}$ to problem (7). In (10), the notation $\circ$ in (a) denotes the Hadamard product; the equality (b) is due to $\tilde{p}_{k_0}=\alpha_{k_0}\hat{p}_{k_0}$ and the definition of $\boldsymbol{\alpha}$ in (9); the strict inequality (c) is due to Lemma 3.1 and the fact $\alpha_{k_0}>1$. Therefore, any $\hat{\mathbf{p}}$ satisfying (8) is an optimal solution to problem (7).

[1] Note that such k_0 might not be unique, and we choose any one of them.

The above proof also implies that the point satisfying condition (8) is unique. Otherwise if there exist $\mathbf{p}_1 \neq \mathbf{p}_2$ such that $g_k(\mathbf{p}_1) = g_k(\mathbf{p}_2) = 1$ for all $k \in \mathcal{K}$, and $\sum_{k\in\mathcal{K}} (\mathbf{p}_1)_k = \sum_{k\in\mathcal{K}} (\mathbf{p}_2)_k$, then we can treat $\mathbf{p}_1$ and $\mathbf{p}_2$ as $\hat{\mathbf{p}}$ and $\tilde{\mathbf{p}}$ in the above, and prove $\mathbf{p}_2$ is not feasible. Hence the point that satisfies condition (8) is unique. This completes the proof of Proposition 3.1. ■

Moreover, by using the Schur complement, we know problem (7) is equivalent to the following SDP

$$
\begin{aligned}
\max_{\{\mathbf{p}\}} \quad & \sum_{k\in\mathcal{K}} p_k \\
\text{s.t.} \quad & \mathbf{M}_k(\mathbf{p}) \succeq p_k(1+\xi)\mathbf{h}_{kk}\mathbf{h}_{kk}^{\dagger},\ k \in \mathcal{K}, \\
& 0 \leq p_k \leq P_k,\ k \in \mathcal{K}.
\end{aligned} \tag{11}
$$

Therefore, for some given $\xi = \bar{\xi}$, the feasibility of problem (6) can be checked in polynomial time by solving (11). In particular, if the solution to (11) satisfies all the SINR inequalities in (7) with equality, then problem (6) with $\xi = \bar{\xi}$ is feasible; otherwise problem (6) is infeasible. Combining with the bisection technique, we see the max-min fairness problem (5) in the SIMO case can be solved in polynomial time.

Theorem 3.1: When each transmitter is equipped with a single antenna, the max-min fairness linear transceiver design problem (2) is polynomial time solvable with arbitrary K and L under the (mild) assumption $\min_k \left\{P_k\|\mathbf{h}_{kk}\|^2/\sigma_k^2\right\} \leq R$, where R is a sufficiently large constant.

Proof: We propose the following SDP bisection algorithm (SDPBA) for problem (2). The algorithm is based on the bisection technique, where each step solves an SDP (11).

Algorithm 1: SDPBA for Problem (2)
S1. Initialization: Choose S_ℓ and S_u such that the optimal SINR lies in $[S_\ell, S_u]$ and a tolerance $\epsilon > 0$.
S2. If $S_u - S_\ell \leq \epsilon$, terminate the algorithm; else go to **S3**.
S3. Let $S_{\text{mid}} = (S_\ell + S_u)/2$ and solve an SDP problem (11) to check the feasibility problem of (6) with $\xi = 1/S_{\text{mid}}$. If feasible, set $S_\ell = S_{\text{mid}}$, else set $S_u = S_{\text{mid}}$ and go to **Step 2**.

We now show the polynomial time complexity of the SDPBA. According to the standard analysis of path-following interior-point methods for SDP, the step **S3** in the SDPBA can be finished in $O(K^{6.5}L^{6.5})$

time [12]. As for the initial choices of S_ℓ and S_u, we let

$$S_\ell = 0,\ S_u = \max_{\{\mathbf{p},\mathbf{u}\}} \min_k \left\{ \frac{|\mathbf{u}_k^\dagger \mathbf{h}_{kk}|^2 p_k}{\sigma_k^2 \|\mathbf{u}_k\|^2} \right\} = \min_k \left\{ \frac{P_k |\mathbf{h}_{kk}|^2}{\sigma_k^2} \right\} \leq R.$$

It takes $\log_2 (R/\epsilon)$ iterations to reach tolerance ϵ. Therefore, a total of $O(\log_2 (R/\epsilon)\, K^{6.5} L^{6.5})$ arithmetic operations are needed in the worst case, and the above algorithm has a polynomial time worst case complexity. ■

Theoretically, the SDPBA is a polynomial time algorithm. However it needs to solve a series of SDPs to get the solution of problem (2), therefore still it is computationally intensive. To reduce the computational complexity, we propose to use the ICCAA in [8] to solve the max-min fairness linear transceiver design problem (2)[2].

Specializing the ICCAA [8] to problem (2), we partition the variables in problem (2) into two blocks $\mathbf{p} = (p_1; p_2, ...; p_K)$ and $\mathbf{u} = (\mathbf{u}_1; \mathbf{u}_2; ...\mathbf{u}_K)$. In particular, when $\mathbf{p} = \mathbf{p}^n$ ($n \geq 0$ denotes the iteration index) fixed, we compute the optimal LMMSE receive beamformer $\mathbf{u}^n$ in (4). On the other hand, when $\mathbf{u} = \mathbf{u}^n$ is fixed, we update the power vector $\mathbf{p}^{n+1}$ by solving the following linear program

$$\begin{aligned} \max_{\{\mathbf{p},\theta\}} \quad & \theta \\ \text{s.t.} \quad & \frac{|\left(\mathbf{u}_k^n\right)^\dagger \mathbf{h}_{kk}|^2 p_k - \theta}{\sigma_k^2 \|\mathbf{u}_k^n\|^2 + \sum\limits_{j \neq k} |\left(\mathbf{u}_k^n\right)^\dagger \mathbf{h}_{kj}|^2 p_j} \geq S_{2n},\ k \in \mathcal{K}, \\ & 0 \leq p_k \leq P_k,\ k \in \mathcal{K}, \end{aligned} \tag{12}$$

where S_{2n} is the minimum SINR value among all users at the point $(\mathbf{u}^n,\ \mathbf{p}^n)$. The algorithm is terminated if $S_{2n+2} - S_{2n} \leq \epsilon$. We know from [8] that the minimum SINR sequence generated by the above ICCAA is globally convergent.

IV. Numerical Simulations

To evaluate the effectiveness of the proposed ICCAA, we present some numerical simulations in this section. We consider a SIMO multi-user interference channel with a single antenna at each transmitter and four antennas ($L = 4$) at each receiver. Similar to [8], we generate the channel vectors according to the complex Gaussian distribution, i.e., $\mathbf{h}_{kj} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}), \forall\, k, j \in \mathcal{K}$. The transmit power budget and the noise power are set to be $P_k = 1$ and $\sigma_k^2 = \sigma^2$ for all users, and the SNR is defined as SNR$= -10 \log_{10} \left(\sigma^2\right)$.

[2] We remark that the algorithms in [3]–[5], [10] based on the uplink-downlink duality theory are all designed for the corresponding optimization problems in the downlink channel with the total power constraint. Therefore, they can not be applied to solve problem (2) with *individual power constraints*.

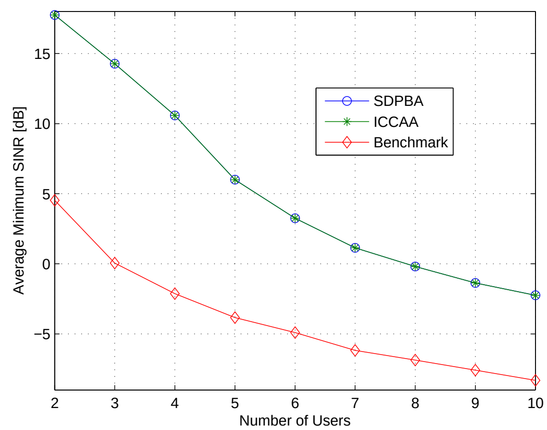

Fig. 1. Average minimum SINR versus the number of users with SNR $= 15$ dB.

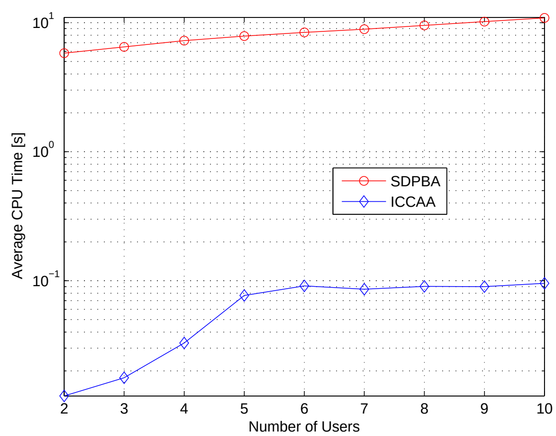

Fig. 2. Average CPU time versus the number of users with SNR $= 15$ dB.

In our simulations, we adopt the so-called channel matched beamformer ($\mathbf{u}_k = \mathbf{h}_{kk}$, $p_k = P_k$) as the benchmark, where each user selfishly maximizes its own received signal power by matching the direct-link channel and transmitting full power. We also use the global optimal solution generated by the SDPBA (Algorithm 1) as the upper bound. We initialize the ICCAA to the channel matched beamformer, and use CVX [13] to solve the SDP problem (11). Each curve in the figures is averaged over 200 independent channel realizations.

Fig. 1 plots the average minimum SINR performance comparison of the proposed algorithms (SDPBA and ICCAA) and the benchmark versus different number of users.This plot shows that the proposed algorithms significantly outperform the benchmark in terms of the achieved minimum SINR value. We can also observe from them that the ICCAA is always able to find the global optimal solution to problem (2). This is not surprising, since we have already shown that problem (2) is polynomial time solvable (Theorem 3.1) and the ICCAA is globally convergent [8].

The average CPU time comparison of the SDPBA and the ICCAA versus different number of users is illustrated as Fig. 2. We can see from it that the ICCAA takes substantially less average CPU time than the SDPBA. This is due to the fact that in each update of the transmit power, one SDP needs to be solved in the SDPBA while one linear program is solved in the ICCAA.

V. Conclusion

In this letter, we consider the max-min fairness linear transceiver design problem for a multi-user SIMO interference channel. A major design challenge is to find the globally optimal transmit/receive beamforming strategy. We show in this letter that, when each transmitter is equipped with a single antenna and each receiver is equipped with multiple antennas, the max-min fairness linear transceiver design problem is polynomial time solvable. This result is in sharp contrast to the corresponding problem for a MIMO interference channel, where each transmitter and receiver are equipped with multiple antennas. In addition, we also propose a low-complexity algorithm, ICCAA, to solve the linear transceiver design problem. Numerical simulations demonstrate that the proposed ICCAA always solves the problem to global optimality with significantly less CPU time compared to the SDPBA.

References

[1] D. Gerlach and A. Paulraj, "Adaptive transmitting antenna methods for multipath environments," in *Proc. IEEE Global Telecommun. Conf. (GLOBECOM)*, Nov. 1994, pp. 425-429.

[2] D. Gerlach and A. Paulraj, "Base station transmitting antenna arrays for multipath environments," *Signal Process.*, vol. 54, no.1, pp. 59-73, Otc., 1996.

[3] H. Boche and M. Schubert, "Resource allocation in multiantenna systems–achieving max-min fairness by optimizing a sum of inverse SIR," *IEEE Trans. Signal Process.,* vol. 54, no. 6, pp. 1990-1997, Jun. 2006.

[4] M. Schubert and H. Boche, "Solution of the multiuser downlink beamforming problem with individual SINR constraints," *IEEE Trans. Veh. Technol.,* vol. 53, no. 1, pp. 18-28, Jan. 2004.

[5] A. Wiesel, Y. C. Eldar, and S. S. Shitz, "Linear precoding via conic optimization for fixed MIMO receivers," *IEEE Trans. Signal Process.*, vol. 54, no. 1, pp. 161-176, Jan. 2006.

[6] Z.-Q. Luo and S. Zhang, "Dynamic spectrum management: Complexity and duality," *IEEE J. Sel. Topics Signal Process.*, vol. 2, no. 1, pp. 57-73, Feb. 2008.

[7] Y.-F. Liu, Y.-H. Dai, and Z.-Q. Luo, "Coordinated beamforming for MISO interference channel: Complexity analysis and efficient algorithms," *IEEE Trans. Signal Process.,* vol. 59, no. 3, pp. 1142-1157, Mar. 2011.

[8] Y.-F. Liu, Y.-H. Dai, and Z.-Q. Luo, “Max-min fairness linear transceiver design for a multi-user MIMO interference channel,” submitted to *IEEE Trans. Signal Process.*

[9] D. P. Palomar, J. M. Cioffi, and M. A. Lagunas, “Joint Tx-Rx beamforming design for multicarrier MIMO channels: A unified framework for convex optimization,” *IEEE Trans. Signal Process.,* vol. 51, no. 9, pp. 2381-2401, Sept. 2003.

[10] E. Visotsky and U. Madhow, “Optimum beamforming using transmit antenna arrays,” in *Proc. IEEE 49th Vehicular Technology Conf., VTC'99,* May 1999, pp.851-856.

[11] G. H. Golub and C. F. Van Loan, *Matrix Computations*, 3rd ed. Baltimore, MD, U.S.A.: The Johns Hopkins Press, 1996.

[12] A. Ben-Tal and A. Nemirovski, *Lectures on Modern Convex Optimization.* Philadelphia, U.S.A.: SIAM-MPS Series on Optimization, SIAM Publications, 2001.

[13] M. Grant, S. Boyd, Y. Ye, *CVX: matlab software for disciplined convex programming,* 2006. [Online]. Available: http://www.stanford.edu/ boyd/cvx/.